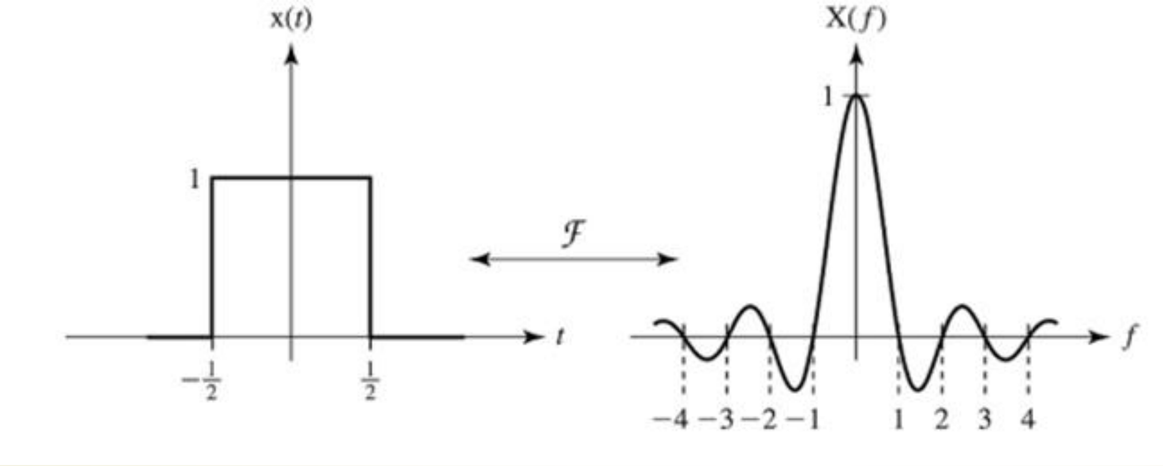
# Análisis de Señales

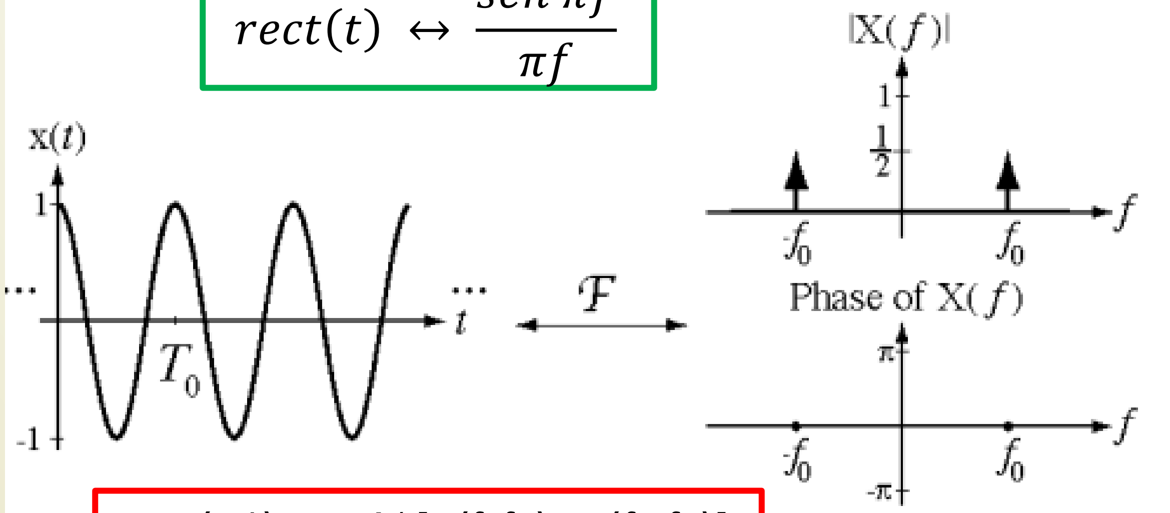
Curso 2021 – Prof. Jorge Runco

Problemas resueltos: Transformada de Fourier en TC

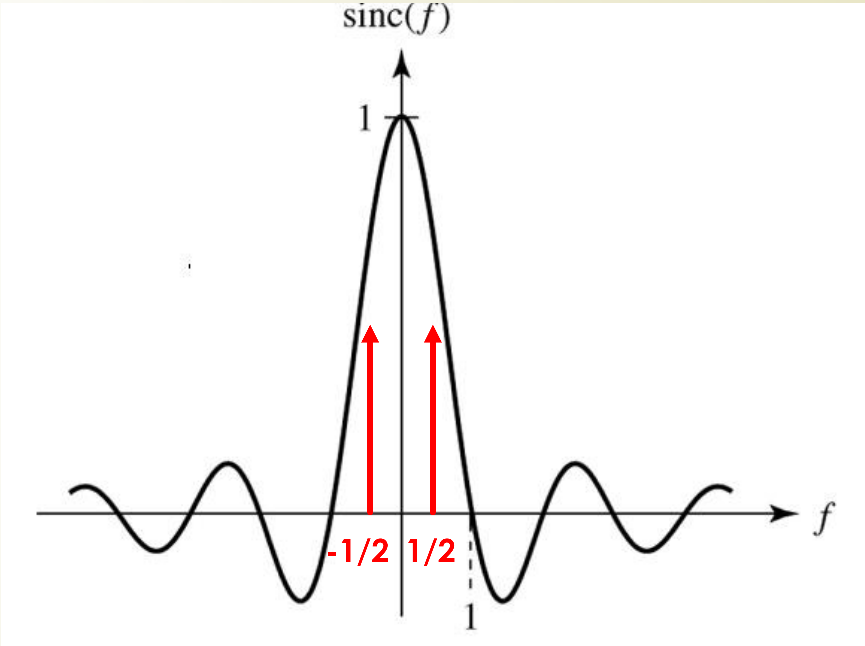
# Ej1)a) y(t)=rect(t)*cos(πt)




$$rect(t) \leftrightarrow \frac{sen\ \pi f}{\pi f}$$

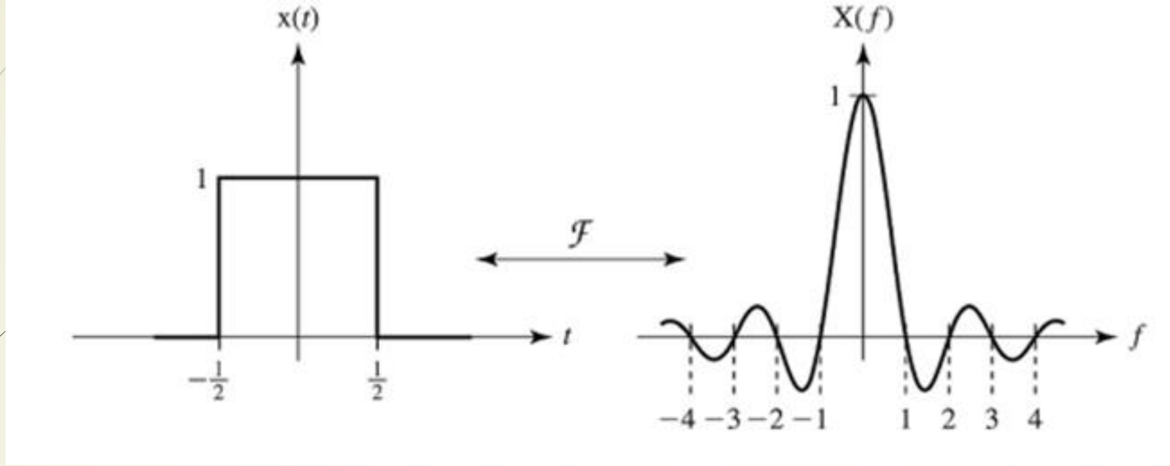



$$\cos(\pi t) \leftrightarrow ½[\delta(f-f_0)+\delta(f+f_0)]$$

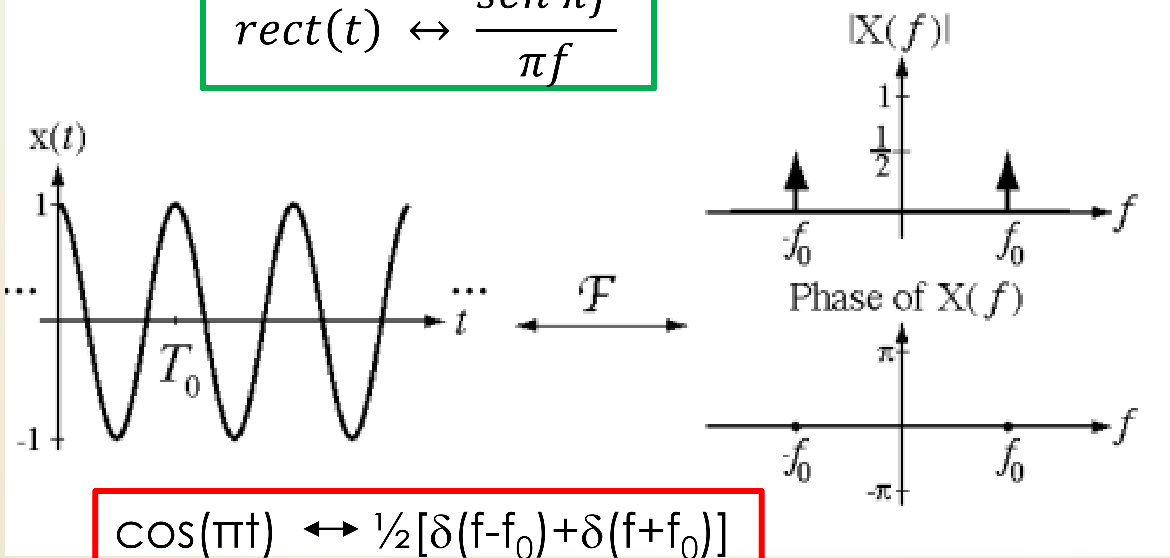


- $y(t) = rect(t) * \cos(\pi t) \quad \leftrightarrow$
- $Y(f) = F\{rect(t)\}. F\{\cos(\pi t)\}$
- $= sinc(f).\frac{1}{2}\left[\delta\left(f-\frac{1}{2}\right)+\delta\left(f+\frac{1}{2}\right)\right]$
- $El\ impulso\ sólo\ tiene\ valor\ en\ f = \frac{1}{2} \rightarrow$
- $\frac{sen\ \pi f}{\pi f} \quad en\ f = \frac{1}{2} \rightarrow \frac{sen\ \pi/2}{\pi/2} = \frac{2}{\pi}$
- $Y(f) = \frac{2}{\pi}.\frac{1}{2}\left[\delta\left(f-\frac{1}{2}\right)+\delta\left(f+\frac{1}{2}\right)\right]$
- $y(t) = \frac{2}{\pi}\cos(\pi t)$ ⟵

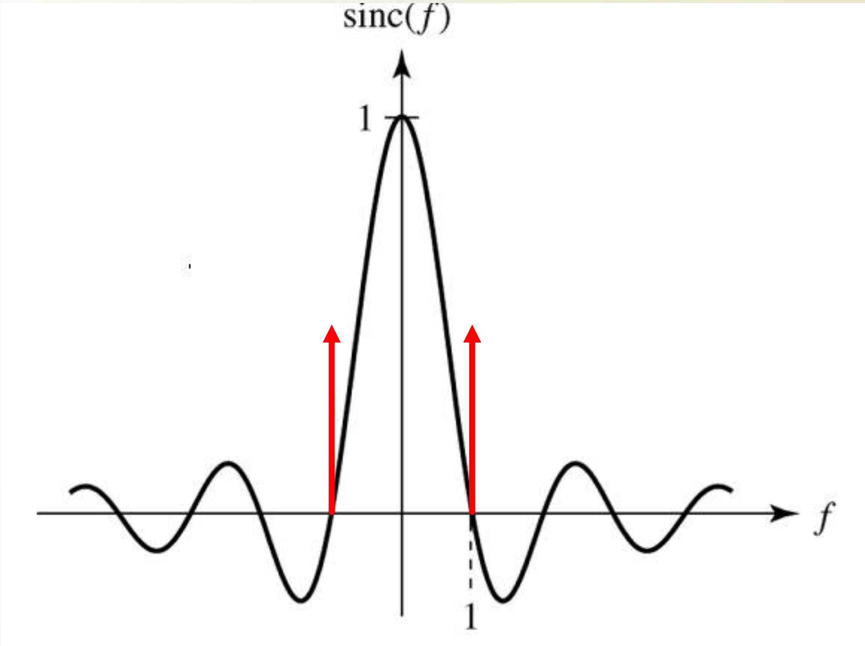
# Ej1)b) y(t)=rect(t)*cos(2πt)




$$rect(t) \leftrightarrow \frac{sen\ \pi f}{\pi f}$$




cos(πt) ↔ ½[δ(f-$f_0$)+δ(f+$f_0$)]




$y(t) = 0$

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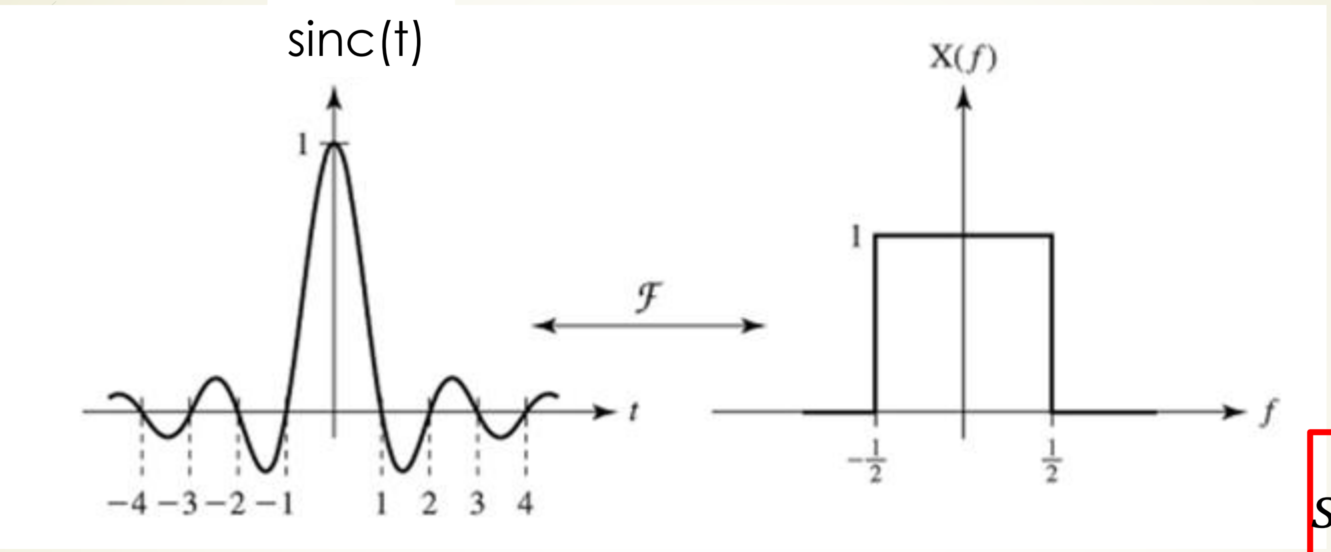
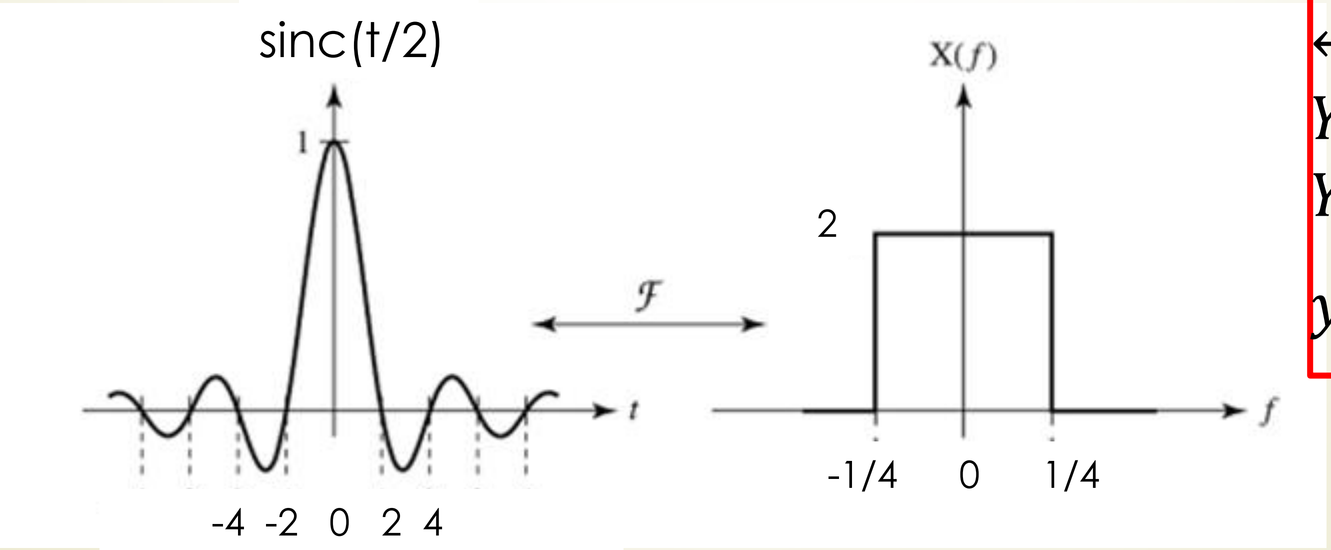
# Ej1)c) y(t)=sinc(t)*sinc(t/2)




$$sinc(t) \ * sinc\left(\frac{t}{2}\right)$$

$$\leftrightarrow rect(f).rect(2f)$$

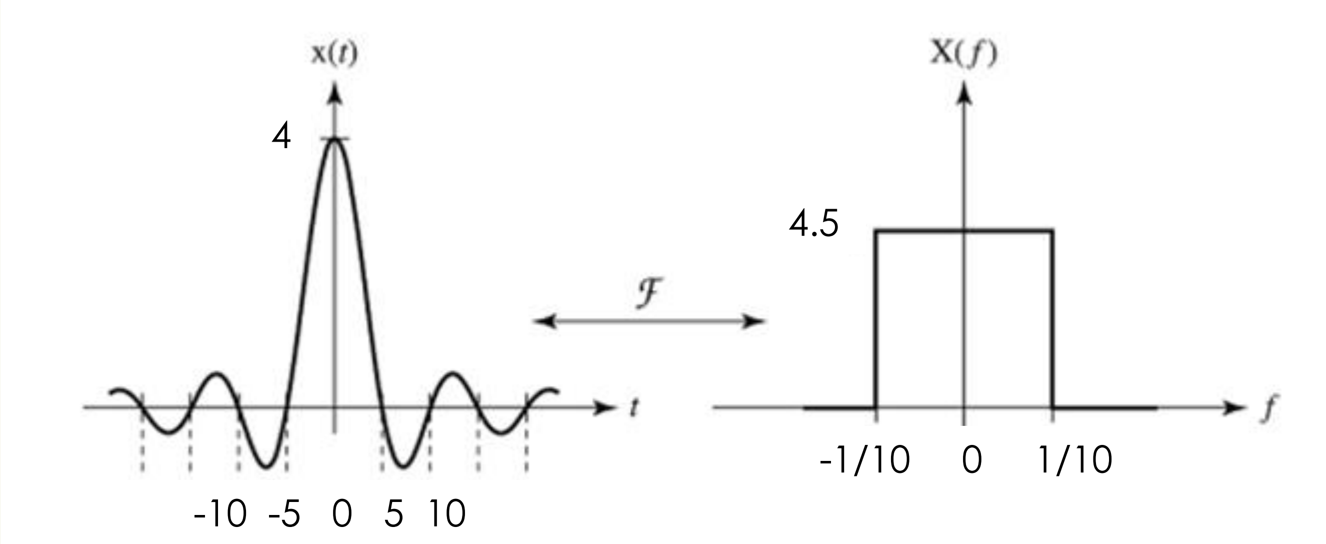
$$Y(f) = rect(f).rect(2f)$$

$$Y(f) = 2\,rect(2f)$$

$$y(t) = sinc\left(\frac{t}{2}\right)$$

# 2)a) x(t)=4 sinc(t/5)

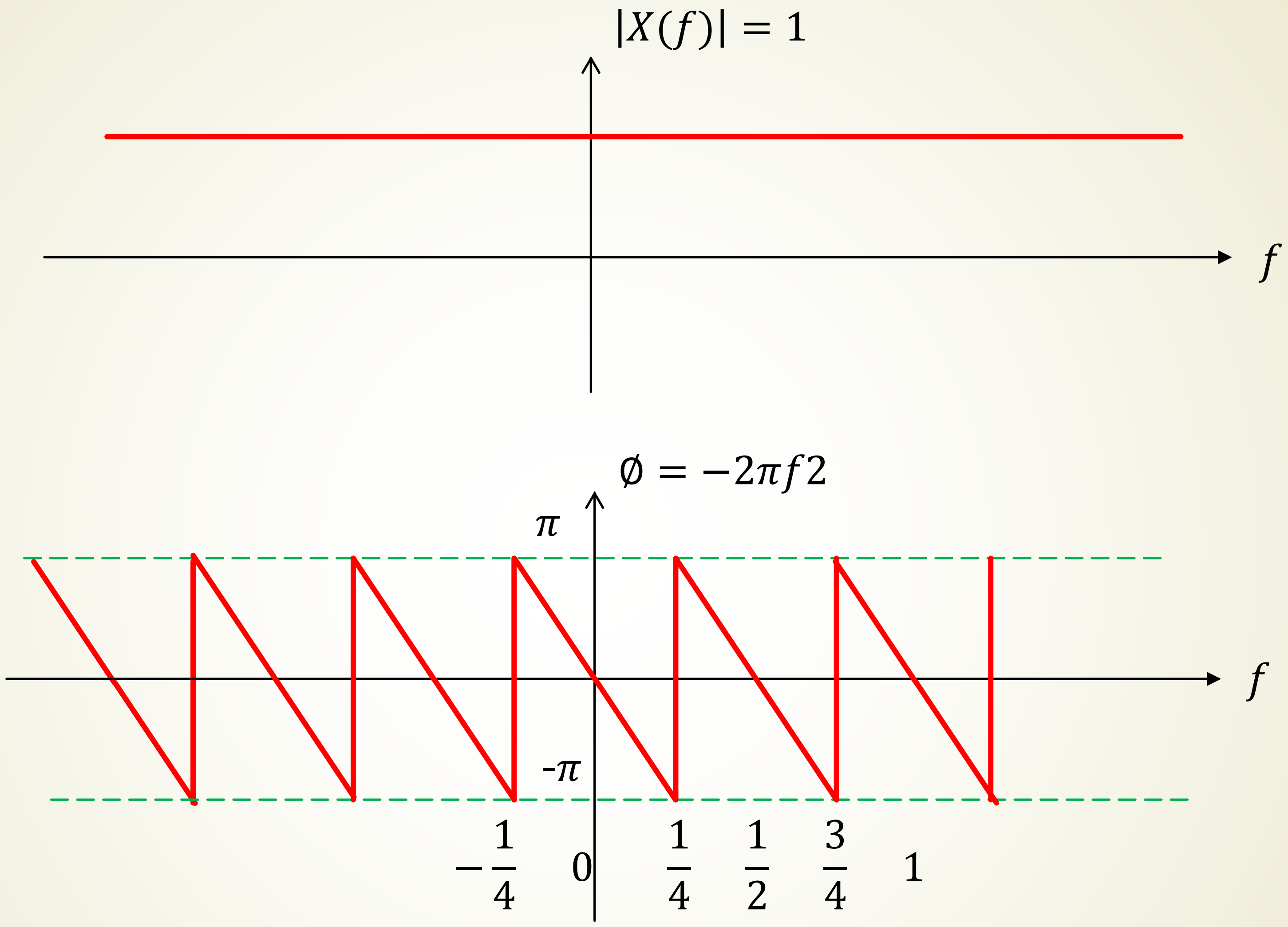
- Teorema de Parseval
- $4\, sinc\left(\frac{t}{5}\right) \leftrightarrow 4.5\, rect(5f)$




- $E = \int_{-\infty}^{+\infty} |X(f)|^2\, df = \int_{-1/10}^{+1/10} (4.5)^2\, df = 400 \,.\frac{2}{10} = 80$

# 3)a) x(t)= $\delta$(t-2)

- $F\{x(t)\} \leftrightarrow X(f)$
- $F\{x(t-t_0)\} \leftrightarrow e^{-j2\pi f t_0} X(f)$
- $F\{\delta(t-2)\} = 1.e^{-j2\pi f 2}$
- $|X(f)| = 1 , \quad e^{-j2\pi f 2} = \cos 2\pi f 2 - j \sin 2\pi f 2$
- $\emptyset = \tan^{-1} - \frac{\sin 2\pi f 2}{\cos 2\pi f 2} = -2\pi f 2$


$|X(f)| = 1$
$f$
$\emptyset = -2\pi f 2$
$\pi$
$f$
$-\pi$
$-\frac{1}{4}$
0
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{3}{4}$
1

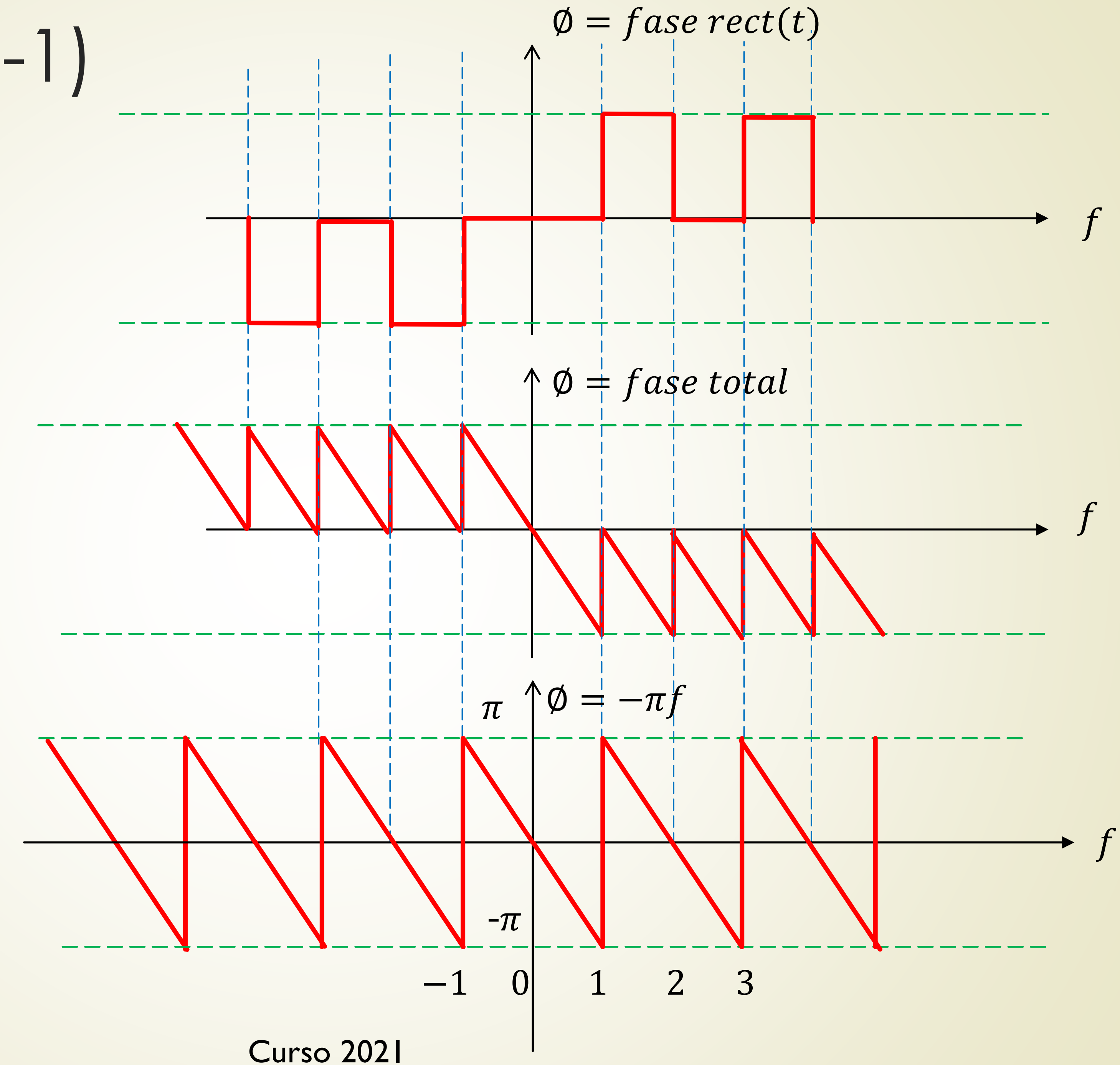
# 3)b) x(t)=u(t) – u(t-1)

$$x(t) = rect\left(t - \frac{1}{2}\right) \overset{F}{\leftrightarrow}$$

$$X(f) = sinc(f).e^{-2\pi f^{1}/_{2}}$$

$$|X(f)| = |sinc(f)|$$

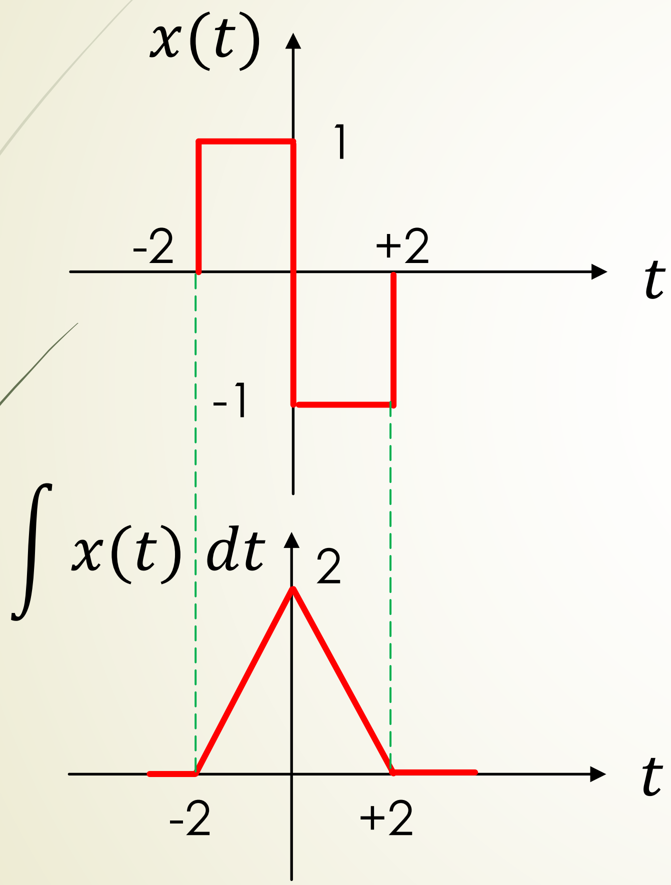
$$\emptyset = fase\, rect(t) - \pi f$$



# 5)

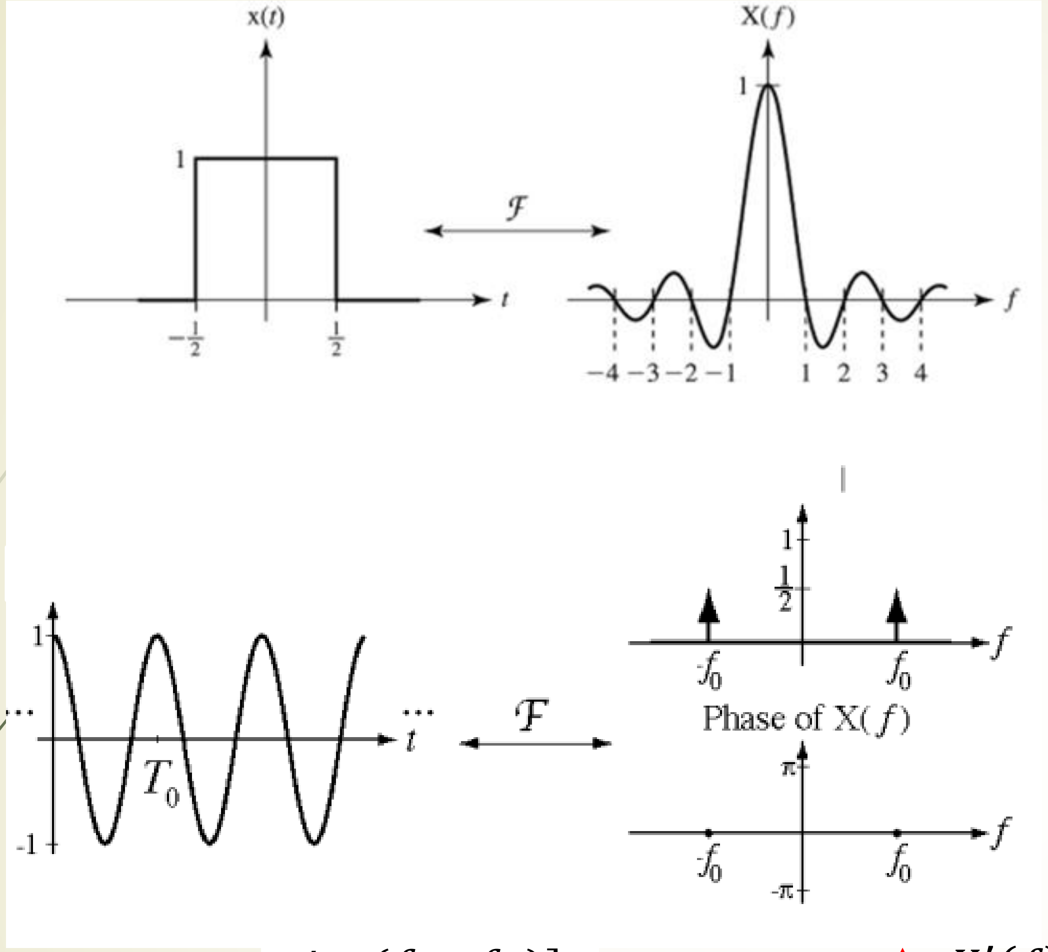
- $a)\ H(f) = sinc(f) \leftrightarrow h(t) = rect(t)$
- $tiene\ \ valor\ para\ t < 0 \rightarrow no\ causal$
- $b)\ H(f) = rect(f) \leftrightarrow h(t) = sinc(t)$
- $tiene\ valor\ para\ t < 0 \rightarrow no\ causal$

7)




- $x(t) = \frac{d}{dt}\left(2\, tri\left(\frac{t}{2}\right)\right)$
- $Utilizando\ propiedad\ de\ escalamiento$
- $2\, tri\left(\frac{t}{2}\right) \overset{F}{\leftrightarrow} 4\, sinc^2\,(2f)$
- $Utilizando\ propiedad\ de\ diferenciación$
- $x(t) \overset{F}{\leftrightarrow} j8\pi f\ \ sinc^2\,(2f)$
- $Si\ aplicamos\ \ escalamiento\ y\ desplazamiento$
- $x(t) \overset{F}{\leftrightarrow} -2\,\frac{\sin 2\pi f}{2\pi f}\,e^{-j2\pi f.1} + 2\,\frac{\sin 2\pi f}{2\pi f}\,e^{+j2\pi f.1}$
- $= 2.2j\,\frac{\sin 2\pi f}{2\pi f}\,\left(\frac{e^{-j2\pi f.1} - e^{+j2\pi f.1}}{2j}\right)$
- $= 4j\,\frac{\sin 2\pi f}{2\pi f}\,.\,\frac{\sin 2\pi f}{2\pi f}\,.\,2\pi f$
- $= j8\pi f\ sinc^2(2f)$

# 8)




- $x'(t) = x(t).\cos 2\pi f_0 t$
- *Si multiplicamos en un dominio*
- *convolucionamos* en el otro
- $X'(f) = X(f) * \frac{1}{2}\,[\delta(f - f_0) + \delta(f + f_0)]$
- *Usaremos* $X(f) = sinc(f)$
- *Aplicando propiedad de modulación*
- $X'(f) = \frac{1}{2}[\,sinc(f - f_0) + sinc(f + f_0\,)]$