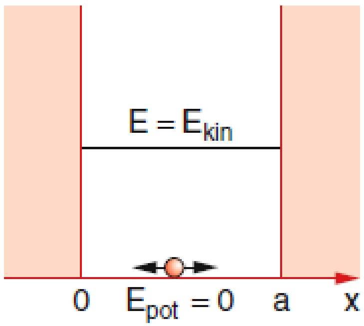
Pozo de potencial unidimensional infinito

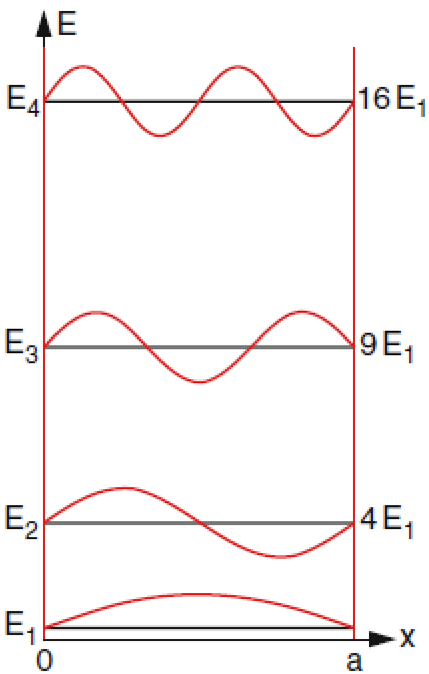
$$E_{\text{pot}}(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \\ \infty & \text{elsewhere} \ . \end{cases}$$

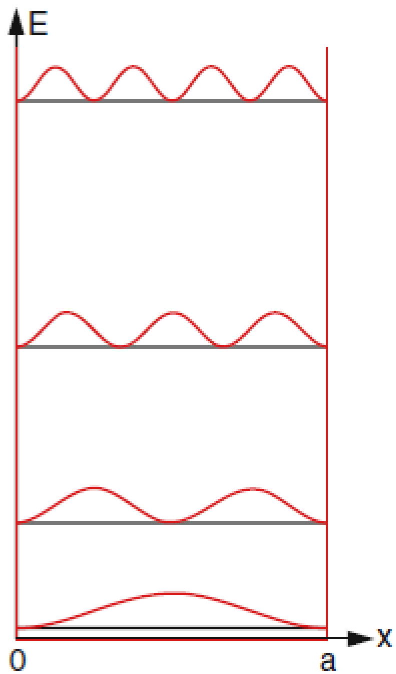
$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + E_{\text{pot}}\psi = E\psi \ .$$

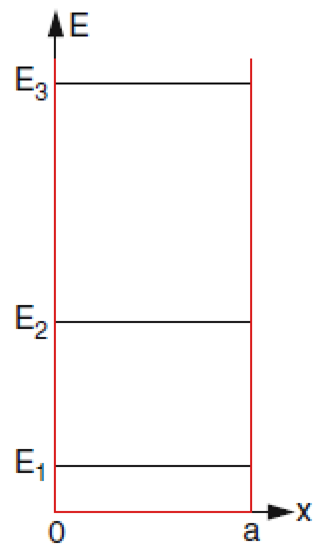
$$\psi(x) = A\,e^{ikx} + B\,e^{-ikx}$$



$$\begin{aligned} E_n &= \frac{p^2}{2m} = \frac{\hbar^2 k_n^2}{2m} \\ &= \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}n^2 \quad (n = 1, 2, 3, \ldots) \end{aligned}$$







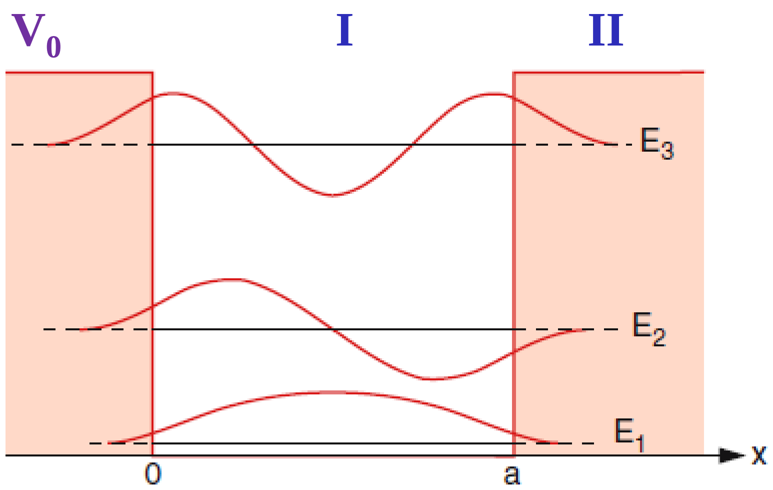
$$E_1 = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}$$

$$E_1 = \frac{h^2}{8ma^2}$$

Pozo de potencial unidimensional finito

$$E_{pot} = \begin{cases} 0 \quad para \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \\ V_0 \; de\; otra\; manera \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E_{\mathrm{pot}}\psi = E\psi \;.$$



$$\psi(x) = A\,\mathrm{e}^{ikx} + B\,\mathrm{e}^{-ikx}$$

$$\psi_{I}(x) = A\cos kx + B\; sen\; kx$$

$$\psi_{II}(x) = Ce^{ik'x} + \mathrm{D}e^{-ik'x}$$

$$\mathrm{k} = \frac{\sqrt{2mE}}{h/2\pi} \qquad \mathrm{k}' = \frac{\sqrt{2\mathrm{m}(\mathrm{E}-V_0)}}{h/2\pi} = \mathrm{i}\kappa$$

$$\kappa = \frac{\sqrt{2\mathrm{m}(V_0-\mathrm{E})}}{h/2\pi}$$

$$\psi_{I}(a/2) = \psi_{II}(a/2)$$

$$\psi'_{I}(a/2) = \psi'_{II}(a/2)$$

Soluciones pares

$$\begin{cases} -A\,k\,\mathrm{sen}\frac{ka}{2} = \text{-C}\,\kappa e^{-\frac{\kappa a}{2}} \\ A\mathrm{cos}\frac{ka}{2} = \mathrm{C}\,e^{-\frac{\kappa a}{2}} \end{cases}$$

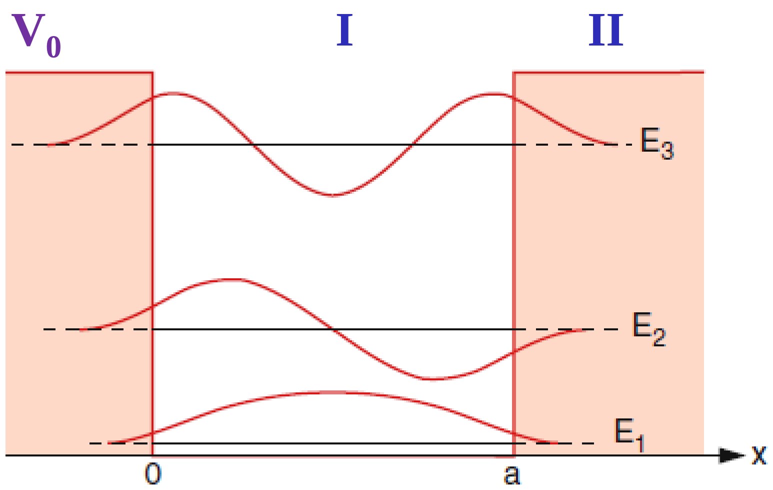
$$\mathrm{k}\; tg\,\frac{ka}{2} = \kappa$$

Pozo de potencial unidimensional finito

$$E_{pot} = \begin{cases} 0 \quad para \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \\ V_0 \; de \; otra \; manera \end{cases}$$

$$\psi_I(x) = A \cos kx \; (B \; sen \; kx)$$

$$\psi_{II}(x) = Ce^{-\kappa x}$$



$$\text{k} = \frac{\sqrt{2mE}}{h/2\pi} \qquad \kappa = \frac{\sqrt{2\text{m}(V_0 - \text{E})}}{h/2\pi}$$

Soluciones pares

$$\text{k} \; tg \frac{ka}{2} = \kappa$$

Soluciones impares

$$\text{k} \; ctg \frac{ka}{2} = -\kappa$$

Definimos:

$$u = \frac{\kappa a}{2} \qquad v = \frac{ka}{2} \qquad u^2 + v^2 = u_0{}^2 \qquad u_0{}^2 = \frac{2mV_0\pi^2 a^2}{h^2}$$

Pozo de potencial unidimensional finito

Soluciones pares $k \, tg \frac{ka}{2} = \kappa$ → $v \, tg \, v = u$

Soluciones impares $k \, ctg \frac{ka}{2} = -\kappa$ → $v \, ctg \, v = -u$

$u^2 + v^2 = u_0^2$

$u = \sqrt{u_0^2 - v^2}$

$$u = \frac{\kappa a}{2} \qquad v = \frac{ka}{2} \qquad u_0^2 = \frac{2mV_0\pi^2a^2}{h^2}$$

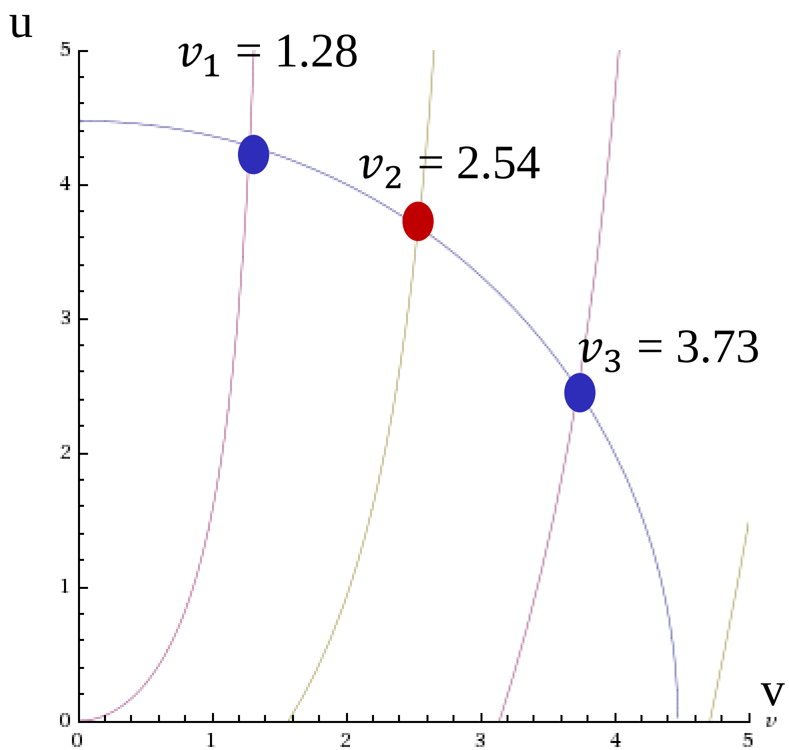
$u_0^2 = 20$ → $u_0 = 4.47$

Hay una solución v_i en cada intervalo

$$\frac{\pi}{2}(i-1) \leq v_i \leq \frac{\pi}{2} i$$

El número de soluciones , para un dado u_0 será :

$$N = \left[\frac{2u_0}{\pi}\right] + 1$$

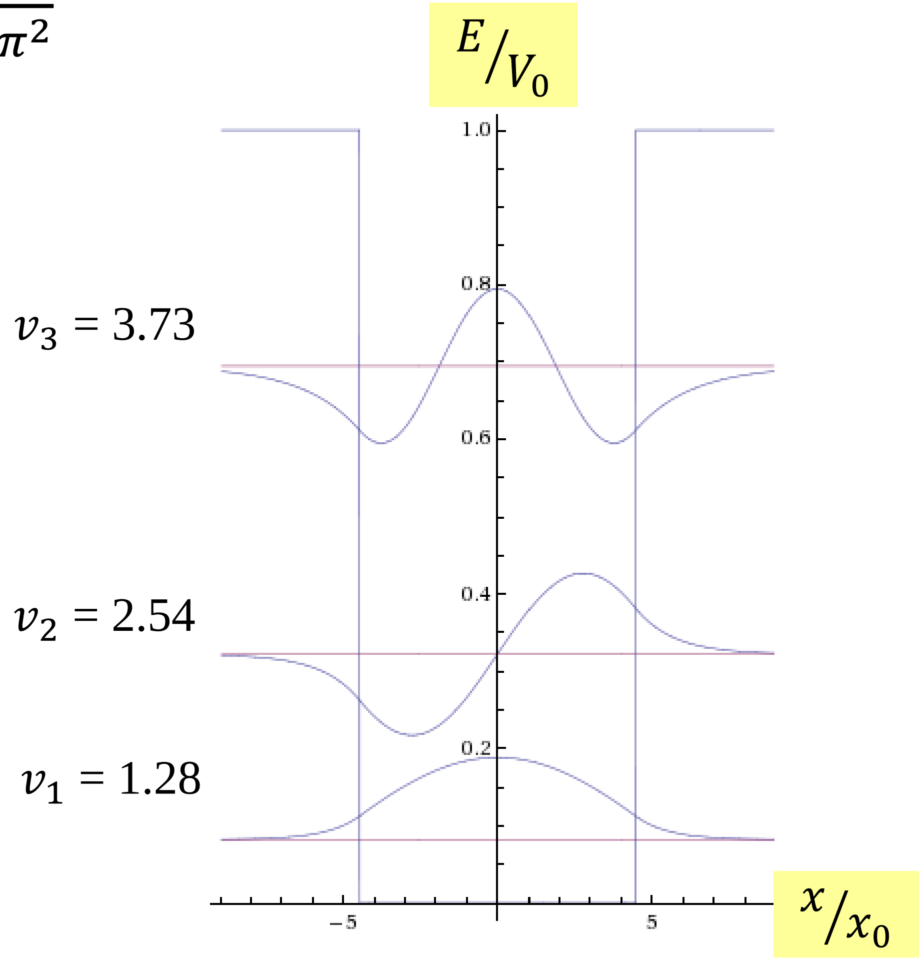


Pozo de potencial unidimensional finito

$$k = \frac{\sqrt{2mE}}{h/2\pi} \qquad \nu = \frac{ka}{2} \qquad E_n = \frac{h^2 {\nu_n}^2}{2ma^2\pi^2}$$

$${u_0}^2 = \frac{2mV_0\pi^2 a^2}{h^2} \qquad \frac{E_n}{V_0} = \frac{{\nu_n}^2}{{u_0}^2}$$

$$x_0 = \frac{h}{\sqrt{8\pi^2 m V_0}}$$



Pozo de potencial unidimensional finito

Es posible establecer los valores permitidos de niveles de energía en función de las profundidades del pozo, mediante las siguientes relaciones:

$y = x|\sec(x\pi/2)|$

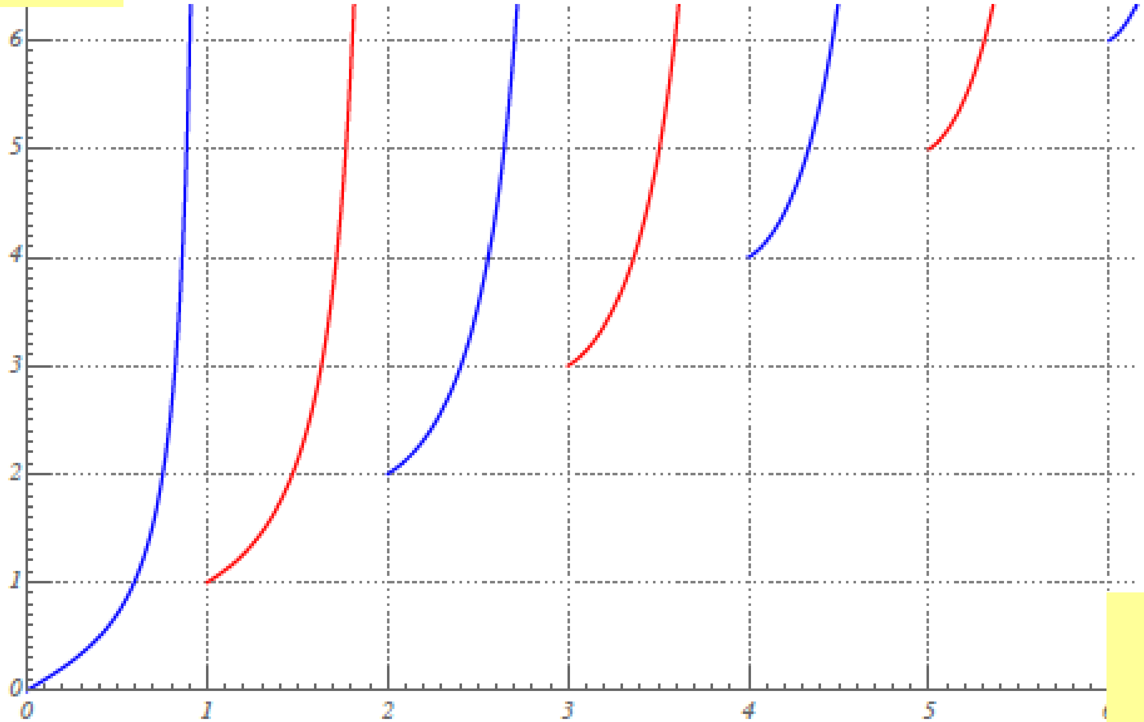
$y = x|\csc(x\pi/2)|$

$$x = \sqrt{E\frac{8ma^2}{h^2}} \qquad y = \sqrt{V_0\frac{8ma^2}{h^2}}$$

Estas ecuaciones son para las funciones de onda de paridad par e impar , respectivamente.
En esas ecuaciones solo deben considerarse las partes de derivadas positivas.
La representación gráfica de los datos (chart) es universal ya que puede ser usada para un análisis gráfico de los niveles de energía permitidos para potenciales de profundidad y ancho arbitrarios.

Pozo de potencial unidimensional finito

$$\sqrt{V_0 \frac{8ma^2}{h^2}}$$

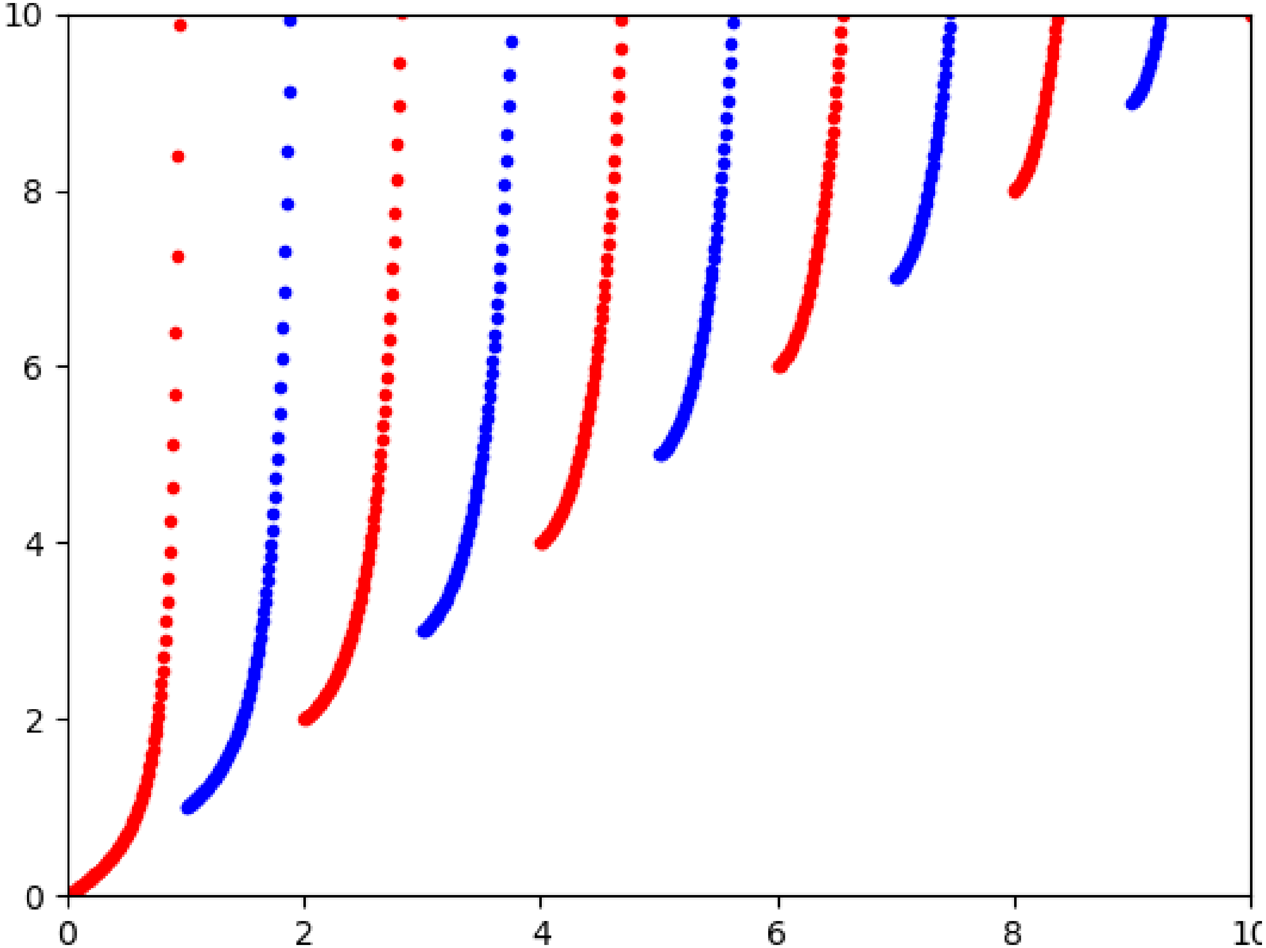


$$\sqrt{E \frac{8ma^2}{h^2}}$$

Chiani, M. (2016). "A chart for the energy levels of the square quantum well".
arXiv:1610.04468 [physics.gen-ph]

Pozo de potencial unidimensional finito

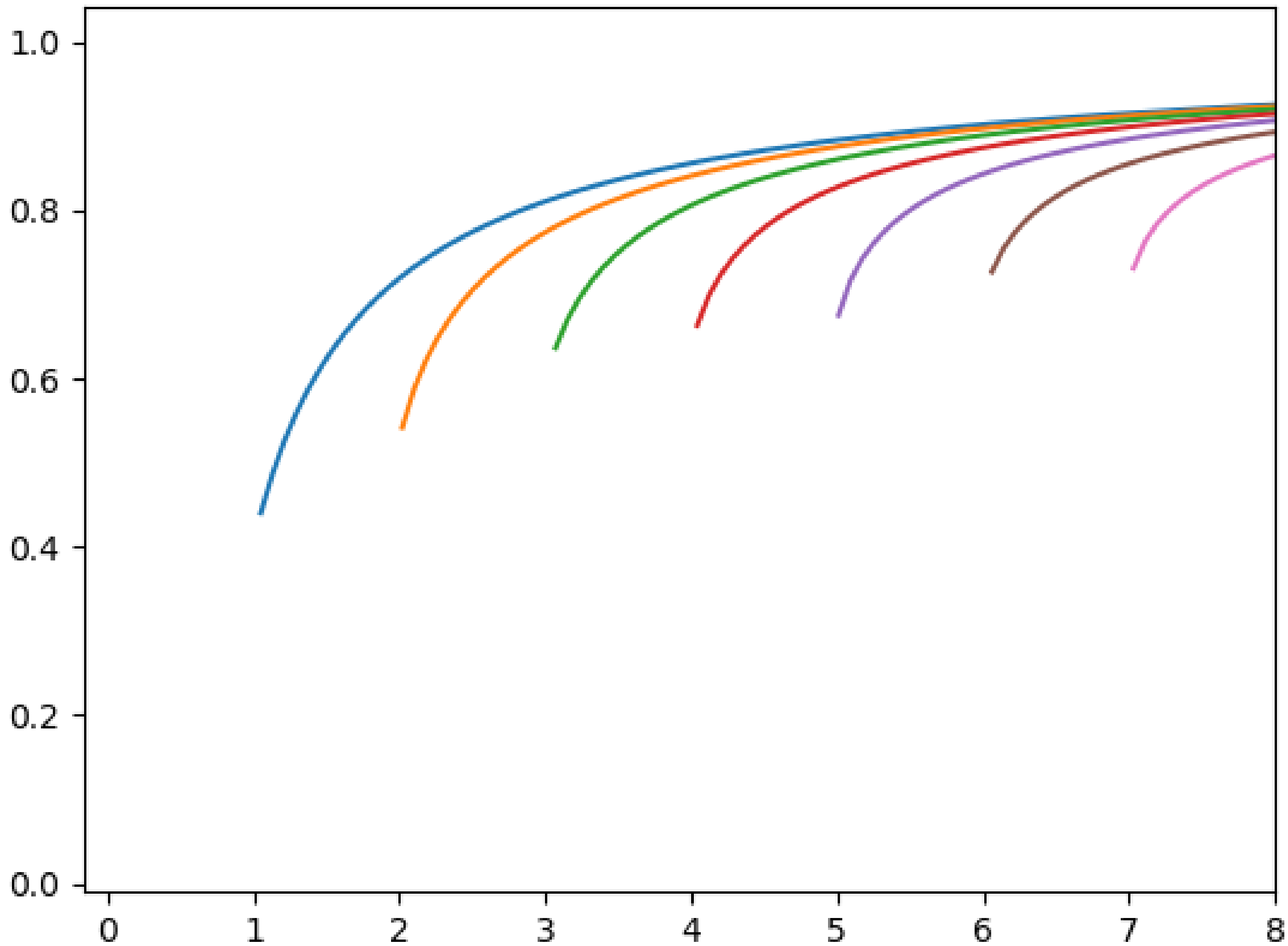
$$\sqrt{V_0 \frac{8ma^2}{h^2}}$$



$$\sqrt{E \frac{8ma^2}{h^2}}$$

Pozo de potencial unidimensional finito

Δx → ΔE



$$\sqrt{V_0 \frac{8ma^2}{h^2}}$$