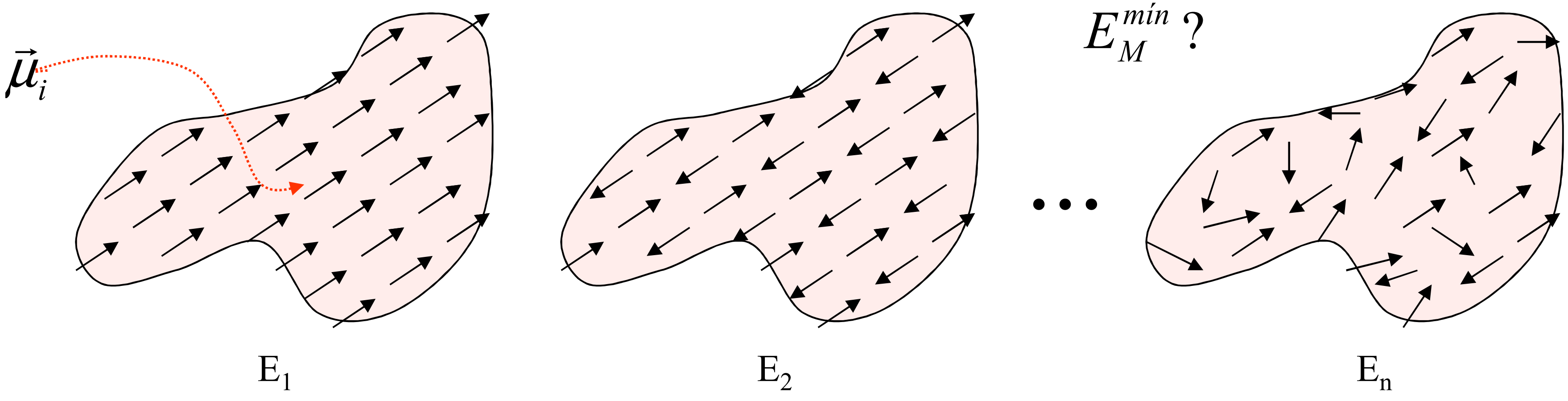
Energía magnetostática – existencia de dominios

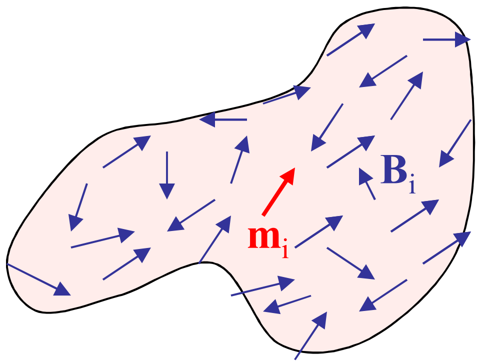
Energía magnetostática

Energía de interación entre los dipolos de un material magnetizado



$$E_M = -\frac{1}{2}\sum_i \vec{\mu}_i \cdot \vec{B}_i = -\frac{\mu_0}{2}\sum_i \vec{\mu}_i \cdot \vec{H}_i = -\frac{\mu_0}{2}\sum_i \vec{M}_i \cdot \vec{H}_i\, V_i \approx -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}_{dip}\, dV$$

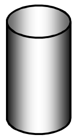
$$\vec{B}_i = \sum_{j\neq i} \vec{B}_j(\vec{r}_i)$$



Evaluación de:

$$E_M = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}\,dV$$

Dado un cuerpo (forma, volumen V, superficie S),



Una distribución de magnetización **M**,

y las ecuaciones de Maxwell:

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} + \partial\vec{B}/\partial t = 0$$

$$\vec{\nabla}\cdot\vec{D} = \rho$$

$$\vec{\nabla}\times\vec{H} - \partial\vec{D}/\partial t = \vec{j}$$

Campos estáticos en ausencia de corrientes:

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = 0$$

además

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$$

$$\vec{H} = -\vec{\nabla}U$$

Potencial escalar

U continuo

$$\vec{\nabla}\cdot\left(-\vec{\nabla}U + \vec{M}\right) = 0$$

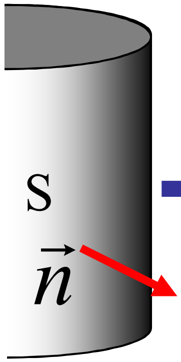
$$\nabla^2 U_{\text{int}} = \vec{\nabla}\cdot\vec{M}$$

$$\nabla^2 U_{out} = 0$$

En el interior de un cuerpo magnetizado uniformemente

$M(\vec{r})$ *dada*

condiciones de contorno



$$U_{\text{int}}\big|_S = U_{ext}\big|_S$$

$$\left.\frac{\partial U_{\text{int}}}{\partial n}\right|_S - \left.\frac{\partial U_{ext}}{\partial n}\right|_S = \vec{M}\cdot\vec{n}$$

+

$$\nabla^2 U_{\text{int}} = \vec{\nabla}\cdot\vec{M}$$

$$\nabla^2 U_{ext} = 0$$

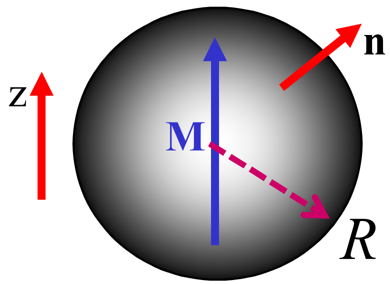
Se puede demostrar que el problema tiene solución única

$$U(\vec{r})$$

$$\vec{H} = -\vec{\nabla} U$$

$$E_M = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}_{dip}\,dV$$

Ejemplo: Esfera de radio R magnetizada uniformemente



$$\vec{\nabla}\cdot\vec{M}=0$$

M uniforme

$$U(\vec{r})=\frac{M_S}{3}\times\begin{cases} z, \ \text{si } r\leq R \\ \dfrac{zR^3}{r^3} \ \text{si } r>R \end{cases}$$

$$\Rightarrow \begin{cases} U_{\text{int}}=\dfrac{M_S}{3}z \\ \vec{H}_{\text{int}}=-\vec{\nabla}U_{\text{int}} \end{cases}$$

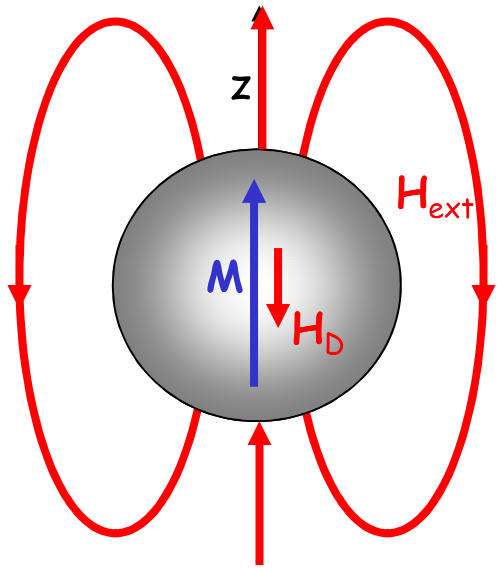
$$\Rightarrow \begin{cases} H_{\text{int}_x}=H_{\text{int}_y}=0 \\ H_{\text{int}_z}=-\dfrac{M_S}{3} \end{cases}$$

$$U_{ext} = \frac{M_S R^3 z}{3r^3} = \frac{M_S R^3 z}{3\left(x^2 + y^2 + z^2\right)^{3/2}}$$

$$H_{ext_x} = \frac{M_S R^3 xz}{r^5}$$

$$H_{ext_y} = \frac{M_S R^3 yz}{r^5}$$

$$H_{ext_z} = \frac{M_S R^3}{r^3}\left(\frac{z^2}{r^2} - \frac{1}{3}\right)$$



$$H_{\mathrm{int}_x} = H_{\mathrm{int}_y} = 0$$

$$H_{\mathrm{int}_z} = -\frac{M_S}{3}$$

energía

$$E_M = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}dV = -\frac{\mu_0}{2}M_S H_{\text{int}_z} V_{esfera} = \frac{\mu_0}{2}M_S \frac{M_S}{3}\frac{4\pi R^3}{3} = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

$$H_{\text{int}_z} = -\frac{M_S}{3}$$

$$E_M = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

generalizando para otras geometrías y distribuciones de la magnetización

$$E_M = C\mu_0 M_S^2$$

$$H_{int_z} = -\frac{M_S}{3}$$

Factor demagnetizante

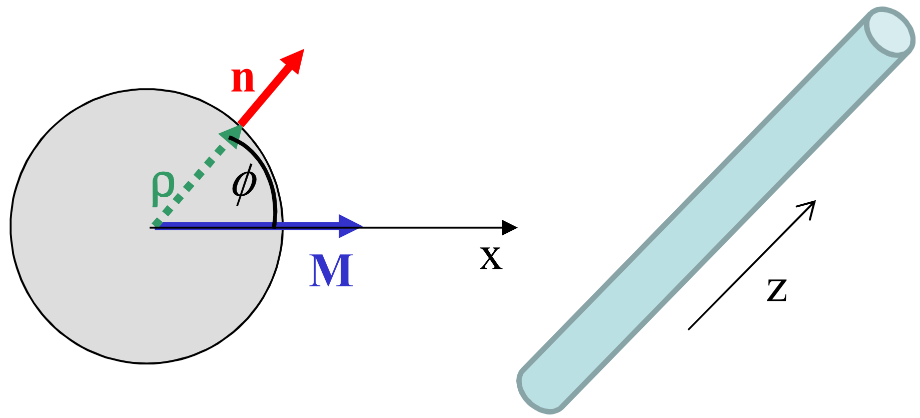
$$H_{int_z} = -N_D M_S$$

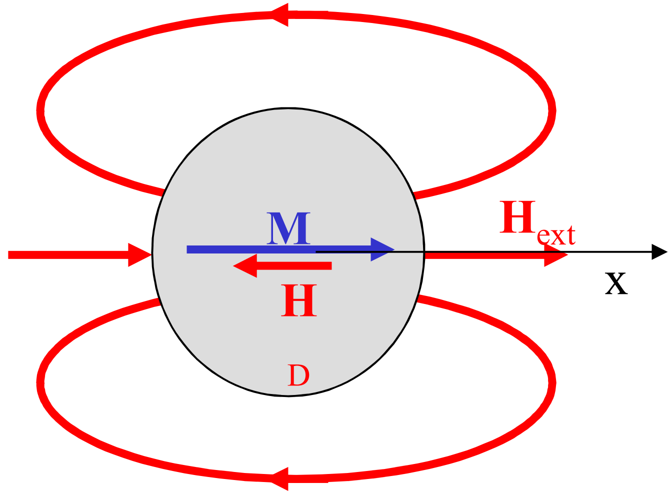
Esfera

$$\begin{cases} \vec{M} = M_S \breve{x} \Rightarrow N_x = 1/3 \\ \vec{M} = M_S \breve{y} \Rightarrow N_y = 1/3 \\ \vec{M} = M_S \breve{z} \Rightarrow N_z = 1/3 \end{cases}$$

$$N_x + N_y + N_z = 1$$

Cilindro infinito magnetizado uniformemente en dirección perpendicular al eje





$$\begin{cases} H_{\text{int}_z} = H_{\text{int}_y} = 0 \\ H_{\text{int}_x} = -\dfrac{M_S}{2} \end{cases}$$

Energía magnetostática por unidad de área ⊥ al eje del cilindro

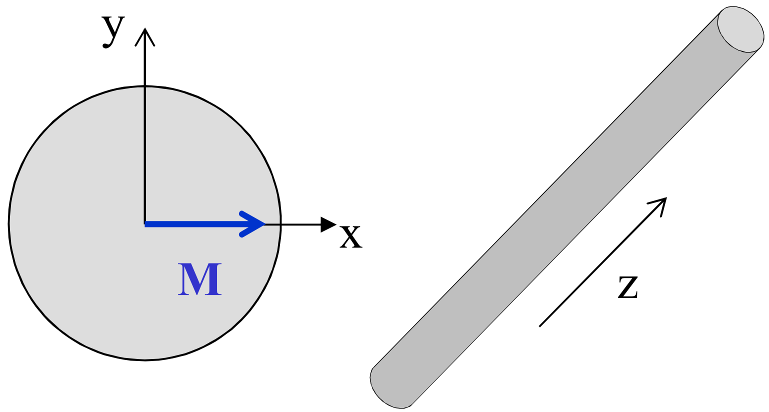
$$H_{\text{int}_x} = -\frac{M_S}{2}$$

$$\varepsilon_M = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}dS = -\frac{\mu_0 \pi R^2}{2}\vec{M}\cdot\vec{H}_{dip} = -\frac{\mu_0}{2}M_S\frac{M_S}{2}\pi R^2 = \frac{\pi R^2 \mu_0 M_S^2}{4}$$

$$\varepsilon_M = \frac{\pi R^2 \mu_0 M_S^2}{4}$$

de la forma

$$\varepsilon_M = C\mu_0 M_S^2$$



$$H_{\mathrm{int}_x} = -\frac{M_S}{2}$$

de la forma

Factor demagnetizante

$$H_{\mathrm{int}_z} = -N_D M_S$$

$$\vec{M} = M_S \breve{x} \Rightarrow N_x = 1/2$$

$$\vec{M} = M_S \breve{y} \Rightarrow N_y = 1/2$$

$$\vec{M} = M_S \breve{z} \Rightarrow N_z = 0$$

$$N_x + N_y + N_z = 1$$

Otros cuerpos magnetizados uniformemente

M uniforme → (Caso general) → $\mathbf{H}_{int}$ NO uniforme

M uniforme → (Superficie de 2do grado (cónicas)) → $\mathbf{H}_{int}$ uniforme

elipsoide

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \quad \longrightarrow \quad \vec{H}_D = -\hat{N}\vec{M}$$

M uniforme

$\vec{H}_D$: Campo demagnetizante

$\hat{N}$: Tensor demagnetizante

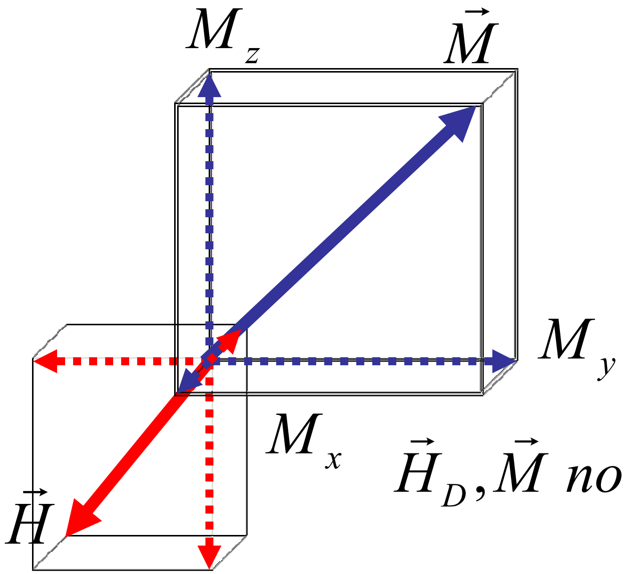
Tensor demagnetizante → Diagonal si los ejes de coordenadas coinciden con los del elipsoide

Tensor demagnetizante → Traza unitaria

$$N_x + N_y + N_z = 1$$

Tensor demagnetizante → Número si además

$$\vec{M} = \begin{cases} M_S \vec{i} \\ M_S \vec{j} \\ M_S \vec{k} \end{cases}$$



$$\vec{H}_D = -\hat{N}\vec{M} \quad \Rightarrow \quad \left.\begin{aligned} H_x &= -N_x M_x \\ H_y &= -N_y M_y \\ H_z &= -N_z M_z \end{aligned}\right\}$$

$\vec{H}_D, \vec{M}$ *no colineales*

$$N_x + N_y + N_z = 1$$

$$M_i, H_i, unif.$$

$$E_M = -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}dV = -\frac{\mu_0}{2}\left(M_x H_x + M_y H_y + M_z H_z\right)V$$

$$E_M = \frac{\mu_0}{2}\left(N_x M_x^2 + N_y M_y^2 + N_z M_z^2\right)V$$

$$E_M^{mín} = 0,\ para\ M = 0$$

Superficies no cuadráticas

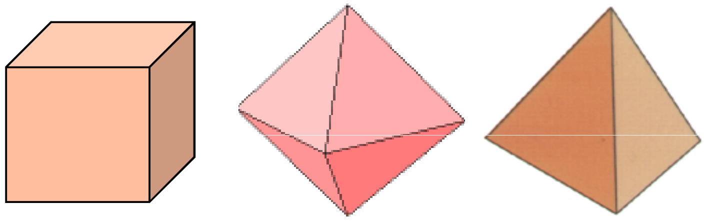
$$E_M = \frac{\mu_0 V}{2}\left(N_x M_x^2 + N_y M_y^2 + N_z M_z^2\right)$$

Válido también para cuerpos con superficies no cuadráticas: cubos, prismas, cilindros, octaedros, etc.

(teorema de Brown-Morrish)

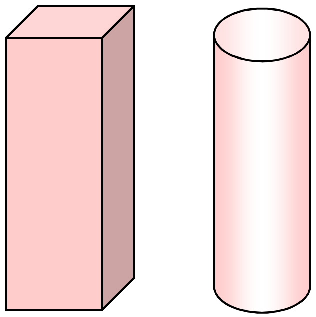
Casos particulares

Cubo, octaaedro, tetraedro



$$N_x = N_y = N_z = 1/3$$

Prisma regular, cilindro

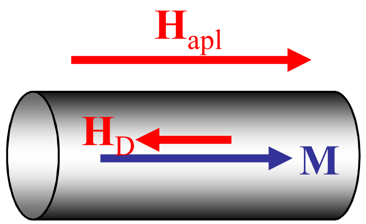


$$N_x = N_y \neq N_z$$

Caso límite

$$N_x = N_y = \frac{1}{2}; \quad N_z = 0$$

campo efectivo



$$H_D = -NM$$

$$H_{ef} = H_{apl} + H_D = H_{apl} - NM$$

Cuando se grafica M vs. H debería usarse como abscisa el H_{ef}

$$M \quad vs \quad H_{ef} = H_{apl} - NM$$

Si M << M_S ⇨ $M = \chi H_{ef}$

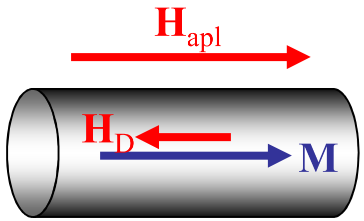
$$H_{ef} = H_{apl} - N\chi H_{ef} \Rightarrow H_{ef} = \frac{H_{apl}}{1+N\chi} \Rightarrow M = \frac{\chi}{1+N\chi} H_{apl}$$

Cuando $\chi << 1$ ⇨ $M \approx \chi H_{apl}$ Independientemente de N

Cuando $\begin{cases} \chi >> 1 \\ N \not<< 1 \end{cases}$ ⇨ $M \approx \frac{1}{N} H_{apl}$ Fuertemente dependiente de N

$\frac{1}{N} \rightarrow \chi_{aparente}$

campo de saturación



$$H_{ef}^{S} = H_{apl} - NM_{S}$$

Ejemplo, Ni

$$M_{S} \approx 4.8x10^{5}\, A/m \approx 0.6\, Tesla$$

$$H_{apl} - NM_{S} > H_{S}^{Ni} \xrightarrow[N\approx 1]{} H_{apl} > 0.6\, Tesla \qquad H_{S}^{Ni} \approx 0.01 - 0.03\, Tesla$$

factores demagnetizantes

Cálculos en prismas

Demagnetizing factors for rectangular ferromagnetic prisms

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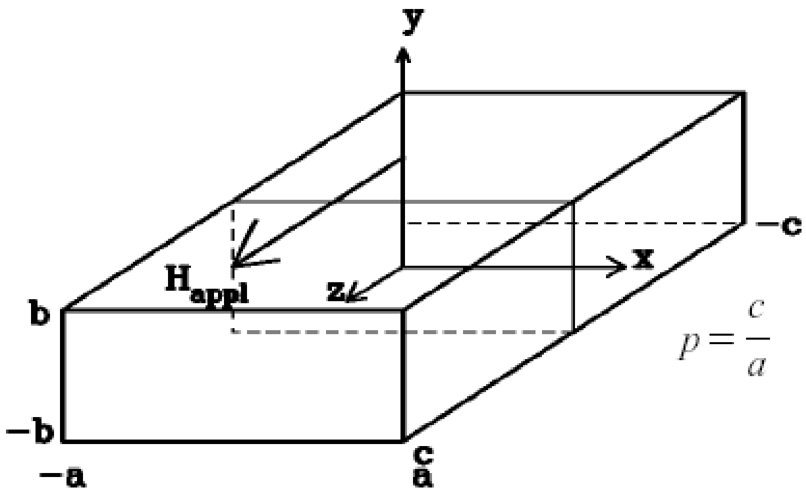


FIG. 1. The coordinate system used in the calculations. Its origin is at the center of the rectangular prism. The field H_{appl} is applied along the z axis.

TABLE I. The demagnetizing factor, D_z^s, of a prolate spheroid and the magnetometric demagnetizing factor, D_z^p, of a square prism, for an aspect ratio, p.

p	D_z^s	D_z^p
2.0	0.17356	0.19832
3.0	0.10871	0.14036
4.0	0.075407	0.10845
5.0	0.055821	0.088316
6.0	0.043230	0.074466
7.0	0.034609	0.064363
8.0	0.028421	0.056670
9.0	0.023816	0.050617
10.0	0.020286	0.045731
11.0	0.017515	0.041705

Factores demagnetizantes– referencias

Fórmulas, tablas y gráficos de factores demagnetizantes, Chen et al. IEEE Trans. Magnetics **27**, 3601-19 (1991)

Campo demagnetizante y medidas magnéticas, J.A. Brug y W.P. Wolf, J.Appl.Phys. **57**, 4685-701 (1985)

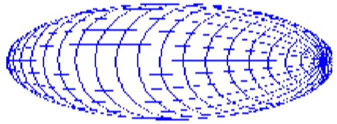
Cálculo de factores demagnetizantes, http://magnet.atp.tuwien.ac.at/dittrich/?http://magnet.atp.tuwien.ac.at/dittrich/content/tools/magnetostatics/streufeld.htm

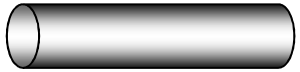
Anisotropía de forma: NP elipsoidales

Elipsoide prolado u oblado

$$a = b < c \Rightarrow N_x = N_y > N_z$$

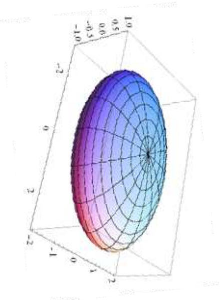
prolado

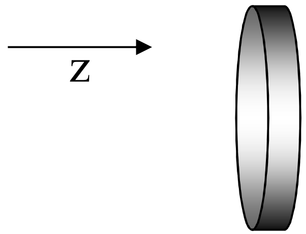




$$a = b > c \Rightarrow N_x = N_y < N_z$$

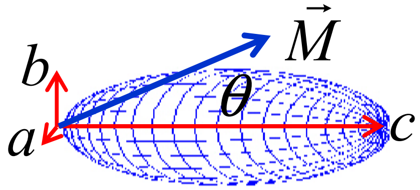
oblado





Anisotropía de forma: NP elipsoidales

Problema: analizar la energía magnetostática de NP elipsoidales



$$a = b < c$$

A partir de la expresión:

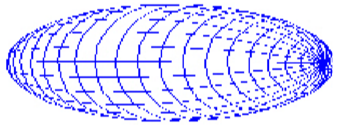
$$E_M = \frac{\mu_0 V}{2}\left(N_x M_x^2 + N_y M_y^2 + N_z M_z^2\right)$$

Comparar con la expresión de la energía de anisotropía uniaxial.
Calcular su valor para una NP de magnetita con forma de elipsoide prolado muy alargado, con:

$$M_S \approx 4.5\times10^5\ A/m$$

Anisotropía de forma: NP elipsoidales

$$a = b < c \Rightarrow N_x = N_y > N_z$$



$$E_M = \frac{\mu_0 V}{2}\left(N_x M_x^2 + N_y M_y^2 + N_z M_z^2\right) \xrightarrow[N_y = N_x]{} \frac{\mu_0 V}{2}\left(N_x\left(M_x^2 + M_y^2\right) + N_z M_z^2\right)$$

$$M_S^2 = M_x^2 + M_y^2 + M_z^2$$

$$E_M = \frac{\mu_0 V}{2}(N_z - N_x)M_z^2 + cte = \frac{\mu_0 V}{2}(N_z - N_x)M_S^2 \cos^2\theta + cte$$

$$E_M = -\frac{\mu_0 V}{2}(N_z - N_x)M_S^2 \sin^2\theta + cte = K_{ME} V \sin^2\theta + cte$$

$$\boxed{E_M = K_{ME} V \sin^2\theta} \qquad \boxed{K_{ME} = \frac{\mu_0}{2}(N_x - N_z)M_S^2}$$

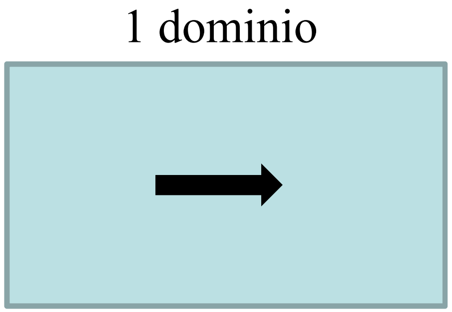
Ejemplo: elipsoide prolado largo de magnetita

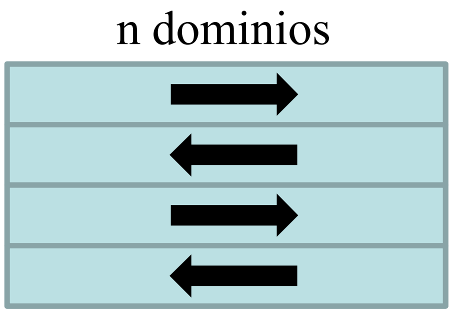
$$K_{ME} = \frac{\mu_0}{2}\left(N_x - N_z\right)M_S^2; \qquad N_x \approx 1/2,\, N_z \approx 0$$

$$M_S \approx 4.5\times10^5 \ A/m$$

$$K_{ME} = \frac{4\pi\times10^{-7}}{2}\frac{1}{2}\left(4.5\times10^5\right)^2 J/m^3 \approx 6.4\times10^4 J/m^3$$

Energía magnetostática - Origen de los dominios



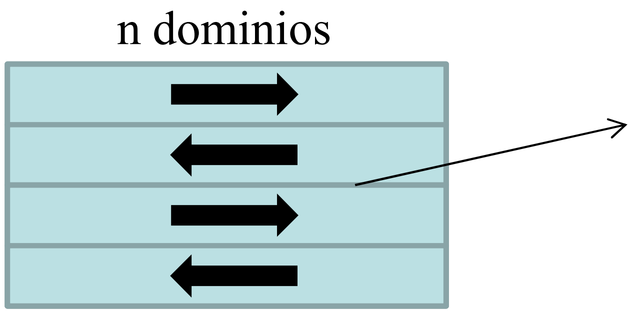


$$E_M(n) = C(n)\mu_0 M_S^2$$

$$\frac{E_M(n)}{E_M(1)} \approx \frac{1}{n}$$

Regla aproximada

Energía de pared de dominios

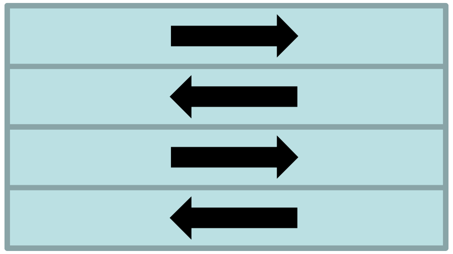


$$\gamma_{PD}\left(J/m^2\right)$$

$$E_{PD} \approx S_{PD}(n)\gamma_{PD}\left(J/m^2\right)$$

Número de dominios en equilibrio

n dominios



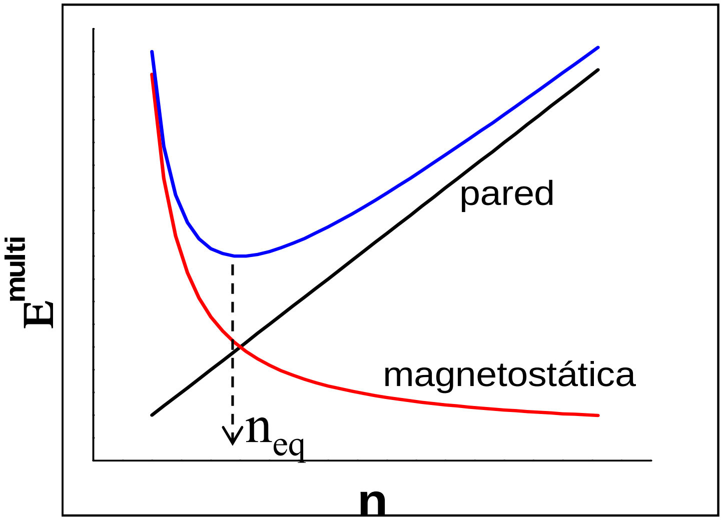
Energía de pared por unidad de área

$$E^{multi} \approx \frac{E_M^{mono}}{n} + S_{PD}(n)\gamma_{PD}$$

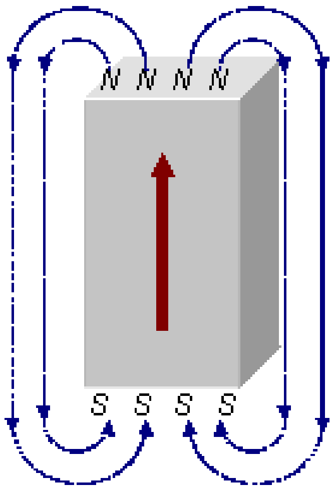
E magnetostática decrece

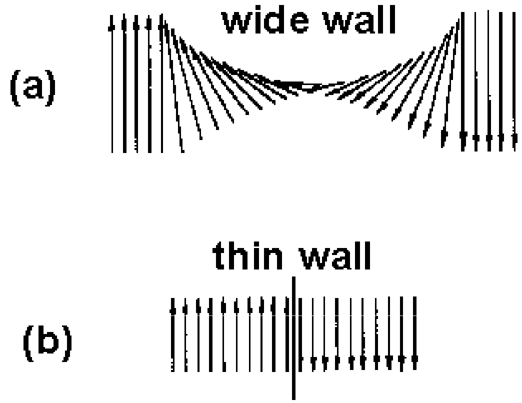
E pared dominios crece

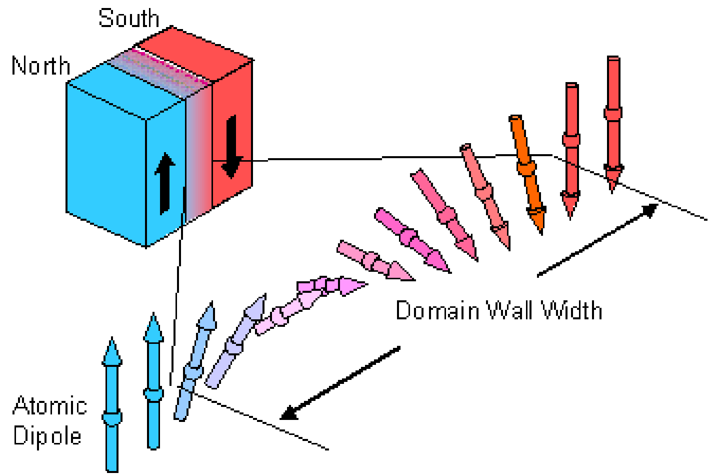
Nro dominios en equilibrio

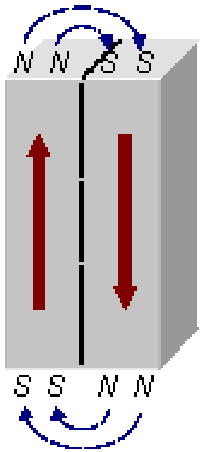


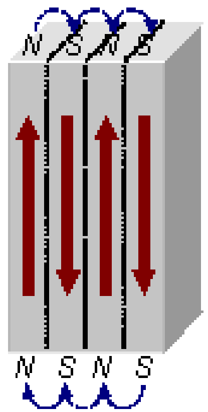
Dominios y paredes de dominio

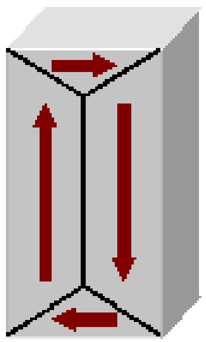


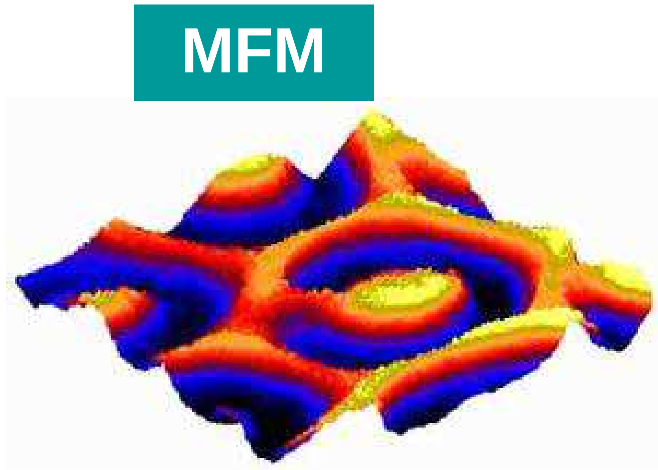






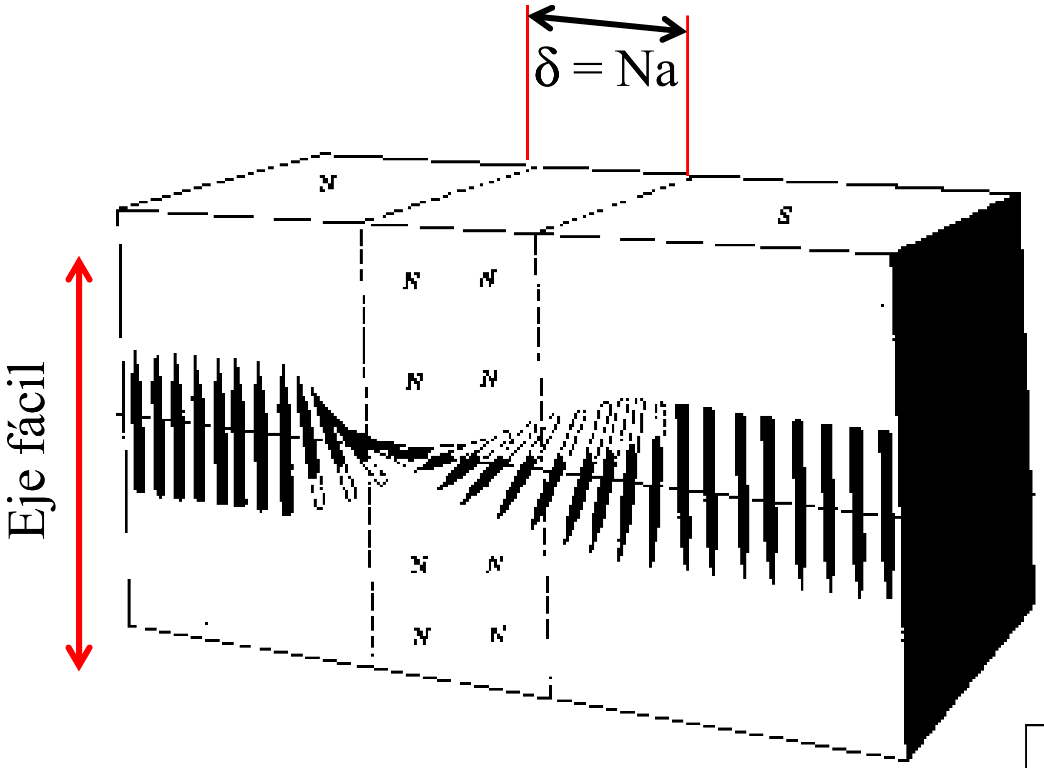




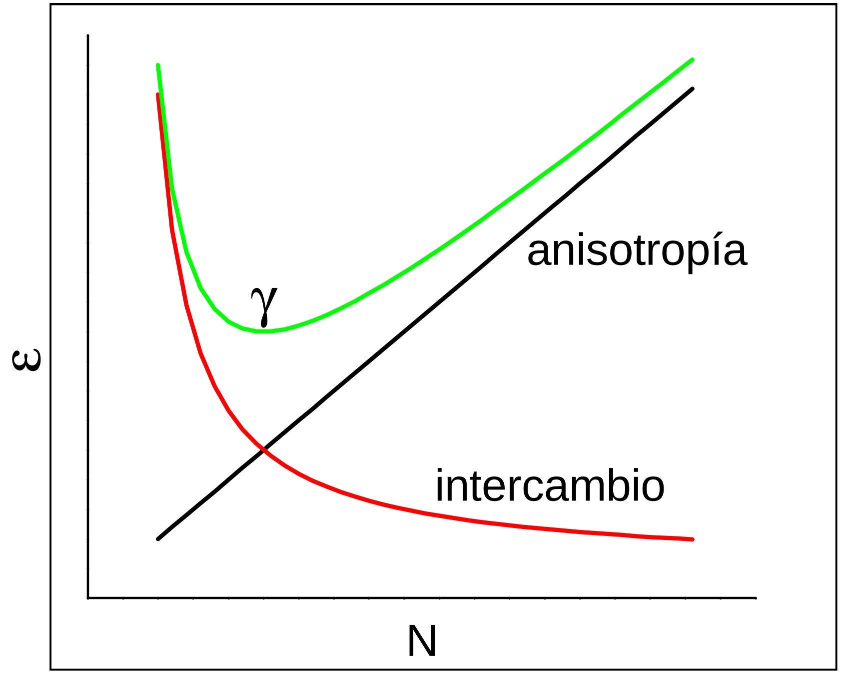


Pseudo-3d MFM image of a $(YSmLaCa)_3$ $(FeGe)_5$ O_{12} magnetic thin film garnet, 4.5 x 4.5 μm^2, domain walls appear dark;

Pared de Bloch de 180°



$$\gamma = \Delta\varepsilon_K + \Delta\varepsilon_J$$



Optimización energía por unidad de área de pared

$$\gamma = \Delta\varepsilon_K + \Delta\varepsilon_J$$

Ancho de la pared $$\delta_{eq} = N_{eq}a = \pi\sqrt{\frac{2Js^2}{Ka}} = \pi\sqrt{\frac{2A}{K}}$$

$$A \approx Js^2 / a$$

Cte de stiffness

$$\gamma_{eq} = 2\pi\sqrt{2KA}$$

$$10^{-12}\, J/m \le A \le 10^{-11}\, J/m$$

$$10^{3}\, J/m^3 \le K \le 10^{6}\, J/m^3$$

$$A = 10^{-11}\ J/m$$

$$K = 10^{3}\ J/m^3 \Rightarrow \delta_{eq} = 444\ nm$$

$$K = 10^{5}\ J/m^3 \Rightarrow \delta_{eq} = 44.4\ nm$$

21st Century Childhood Pranks.

Fin módulo