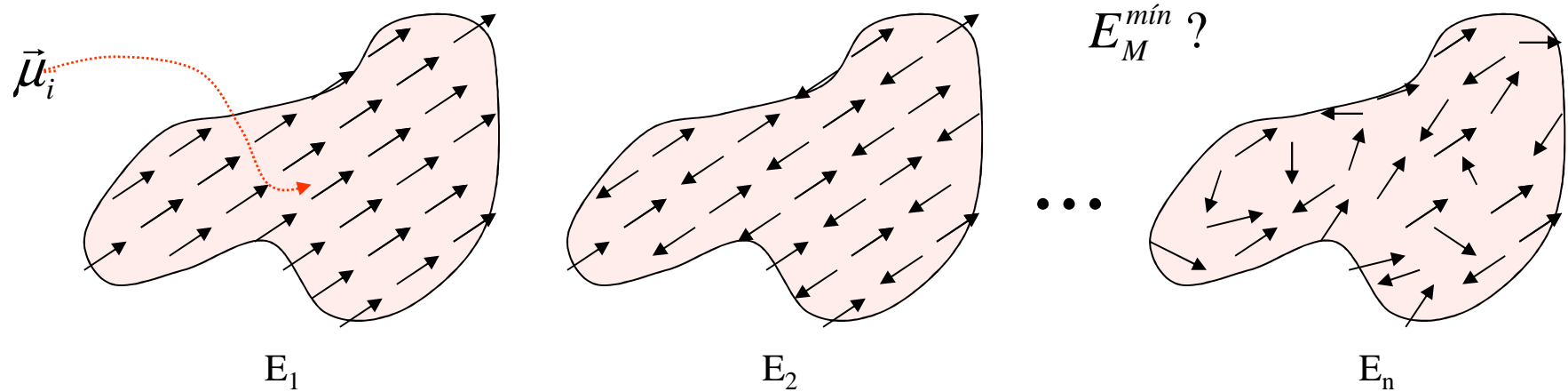


Energía magnetostática – existencia de dominios

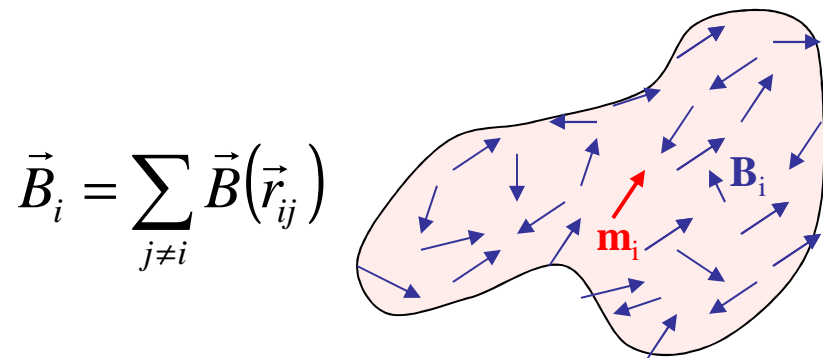
Energía magnetostática

Energía de interacción entre los dipolos de un material magnetizado

$$E_Z = -\vec{\mu}_i \cdot \vec{B}_i$$



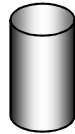
$$E_M = -\frac{1}{2} \sum_i \vec{\mu}_i \cdot \vec{B}_i = -\frac{\mu_0}{2} \sum_i \vec{\mu}_i \cdot \vec{H}_i = -\frac{\mu_0}{2} \sum_i \vec{M}_i \cdot \vec{H}_i V_i \approx -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H}_{dip} dV$$



Evaluación de:

$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV$$

Dado un cuerpo (forma, volumen V , superficie S),



Una distribución de magnetización \vec{M} ,

y las ecuaciones de Maxwell:

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} + \partial \vec{B} / \partial t = 0$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{H} - \partial \vec{D} / \partial t = \vec{j}$$

Campos estáticos en ausencia de corrientes:

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = 0$$

además

$$\vec{H} = -\vec{\nabla} U$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

Potencial escalar

U continuo

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (-\vec{\nabla} U + \vec{M}) = \nabla^2 U + \vec{\nabla} \cdot \vec{M} = 0$$

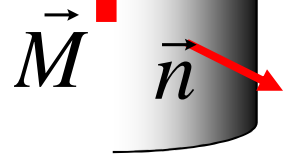
$$\nabla^2 U_{\text{int}} = \vec{\nabla} \cdot \vec{M}$$

$$\nabla^2 U_{\text{out}} = 0$$

En el interior y exterior de un cuerpo magnetizado uniformemente

$M(\vec{r})$ dada

condiciones de contorno



$$U_{\text{int}}|_S = U_{\text{ext}}|_S$$
$$\frac{\partial U_{\text{int}}}{\partial n}\bigg|_S - \frac{\partial U_{\text{ext}}}{\partial n}\bigg|_S = \vec{M} \cdot \vec{n}$$

Se puede demostrar que el problema tiene solución única

$$U(\vec{r})$$

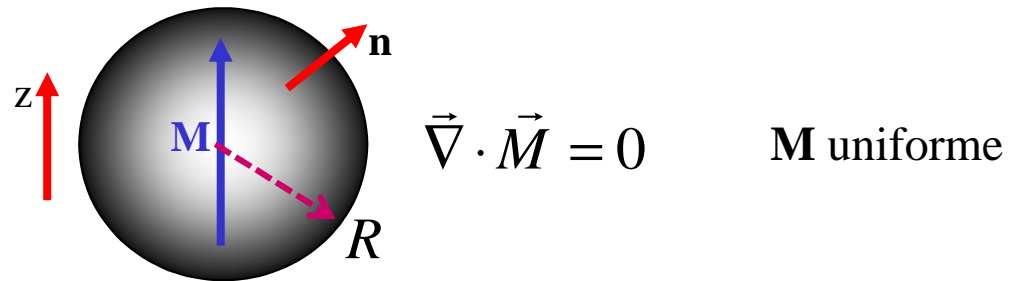
$$\vec{H} = -\vec{\nabla} U$$

$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H}_{\text{dip}} dV$$

+

$$\nabla^2 U_{\text{int}} = \vec{\nabla} \cdot \vec{M}$$
$$\nabla^2 U_{\text{ext}} = 0$$

Ejemplo: Esfera de radio R magnetizada uniformemente

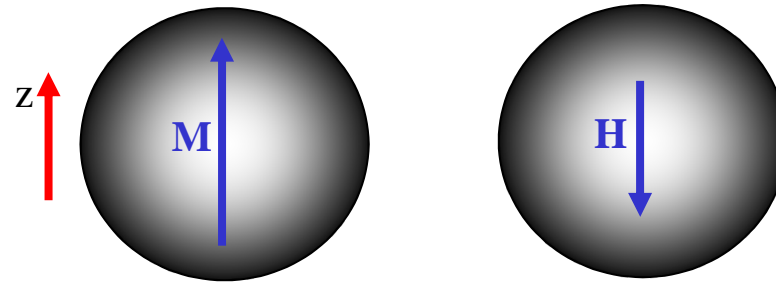


$$U(\vec{r}) = \frac{M_S}{3} \times \begin{cases} z, & \text{si } r \leq R \\ \frac{zR^3}{r^3} & \text{si } r > R \end{cases}$$

$$\begin{cases} H_{\text{int}_x} = H_{\text{int}_y} = 0 \\ H_{\text{int}_z} = -\frac{M_S}{3} \end{cases}$$

Interior

M uniforme



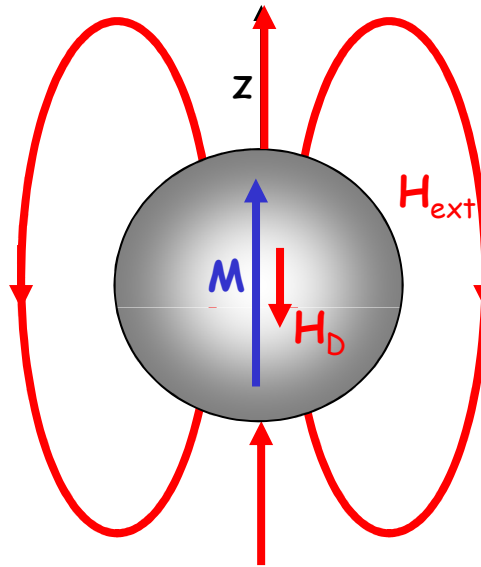
$$\left\{ \begin{array}{l} H_{\text{int}_x} = H_{\text{int}_y} = 0 \\ H_{\text{int}_z} = -\frac{M_S}{3} \end{array} \right.$$

Exterior

$$H_{ext_x} = \frac{M_S R^3 xz}{r^5}$$

$$H_{ext_y} = \frac{M_S R^3 yz}{r^5}$$

$$H_{ext_z} = \frac{M_S R^3}{r^3} \left(\frac{z^2}{r^2} - \frac{1}{3} \right)$$

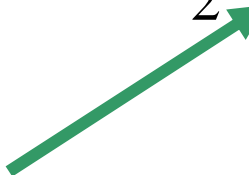


$$H_{int_x} = H_{int_y} = 0$$

$$H_{int_z} = -\frac{M_S}{3}$$

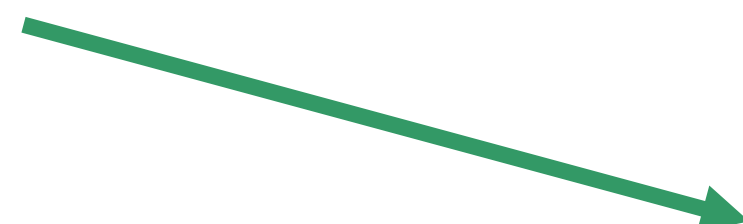
Energía magnetostática

$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV = -\frac{\mu_0}{2} M_S H_{\text{int}_z} V_{\text{esfera}} = \frac{\mu_0}{2} M_S \frac{M_S}{3} \frac{4\pi R^3}{3} = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

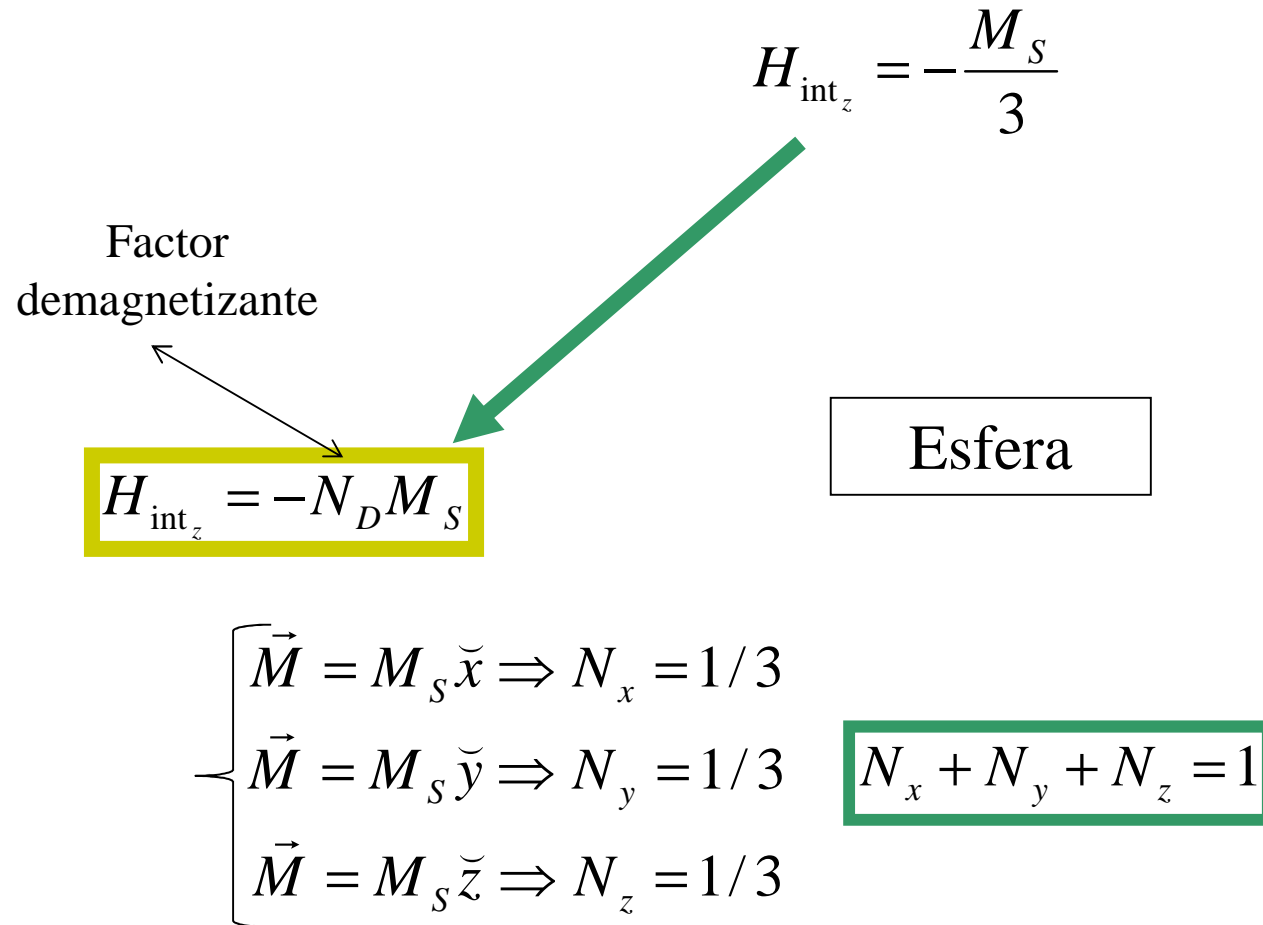
$$H_{\text{int}_z} = -\frac{M_S}{3}$$


$$E_M = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

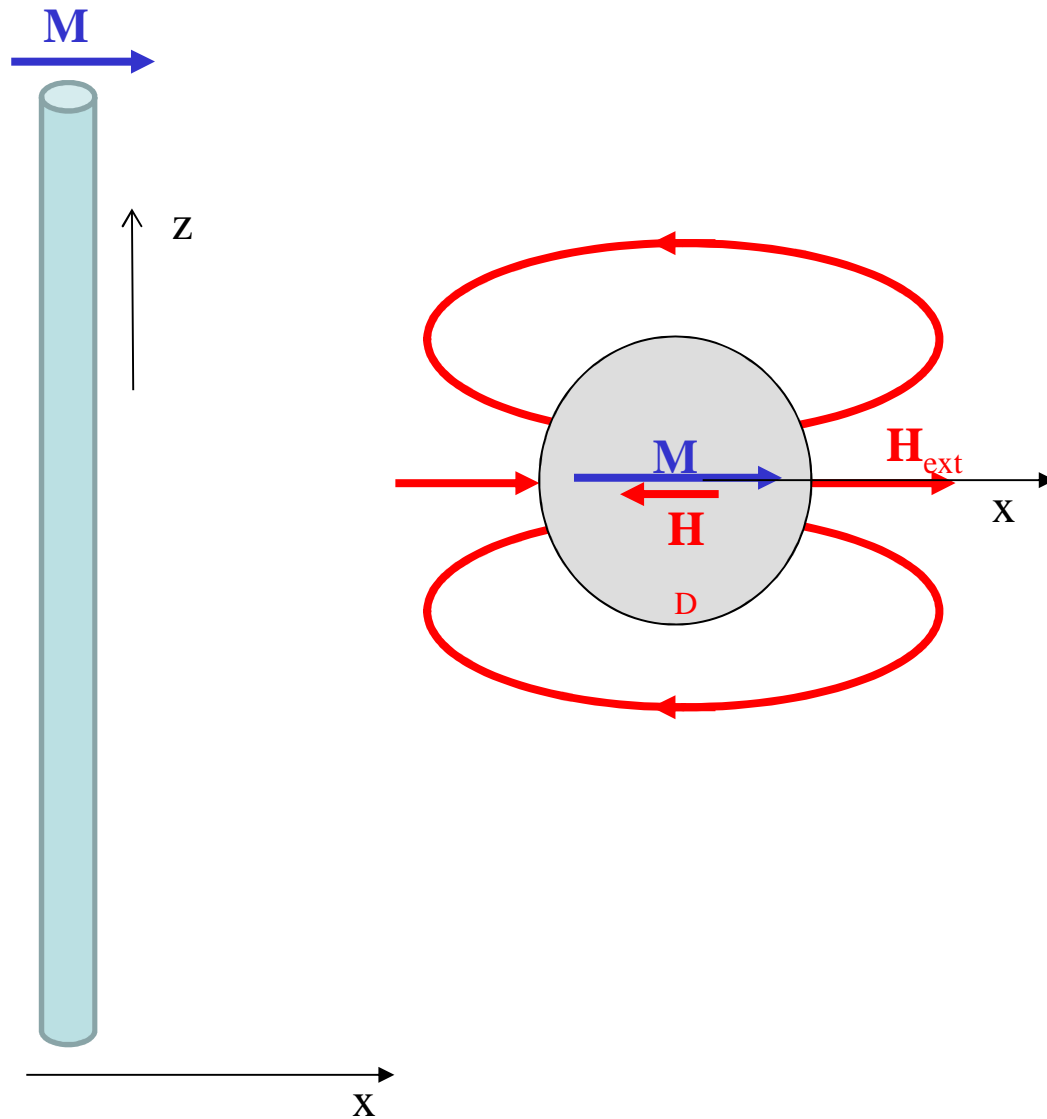
$$\epsilon_M = \frac{E_M}{\text{Vol}} = \frac{2\pi R^3 \mu_0 M_S^2}{9 \times 4\pi R^3 / 3} = \frac{\mu_0 M_S^2}{6}$$

$$\epsilon_M = C \mu_0 M_S^2$$


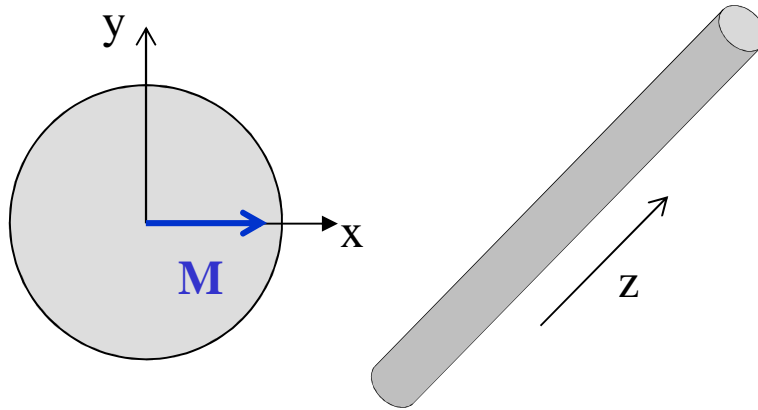
Factor desmagnetizante



Cilindro infinito magnetizado uniformemente en dirección perpendicular al eje



$$\begin{cases} H_{\text{int}_z} = H_{\text{int}_y} = 0 \\ H_{\text{int}_x} = -\frac{M_S}{2} \end{cases}$$



muy largo

$$H_{\text{int}_x} = -\frac{M_S}{2}$$

de la forma

Factor demagnetizante

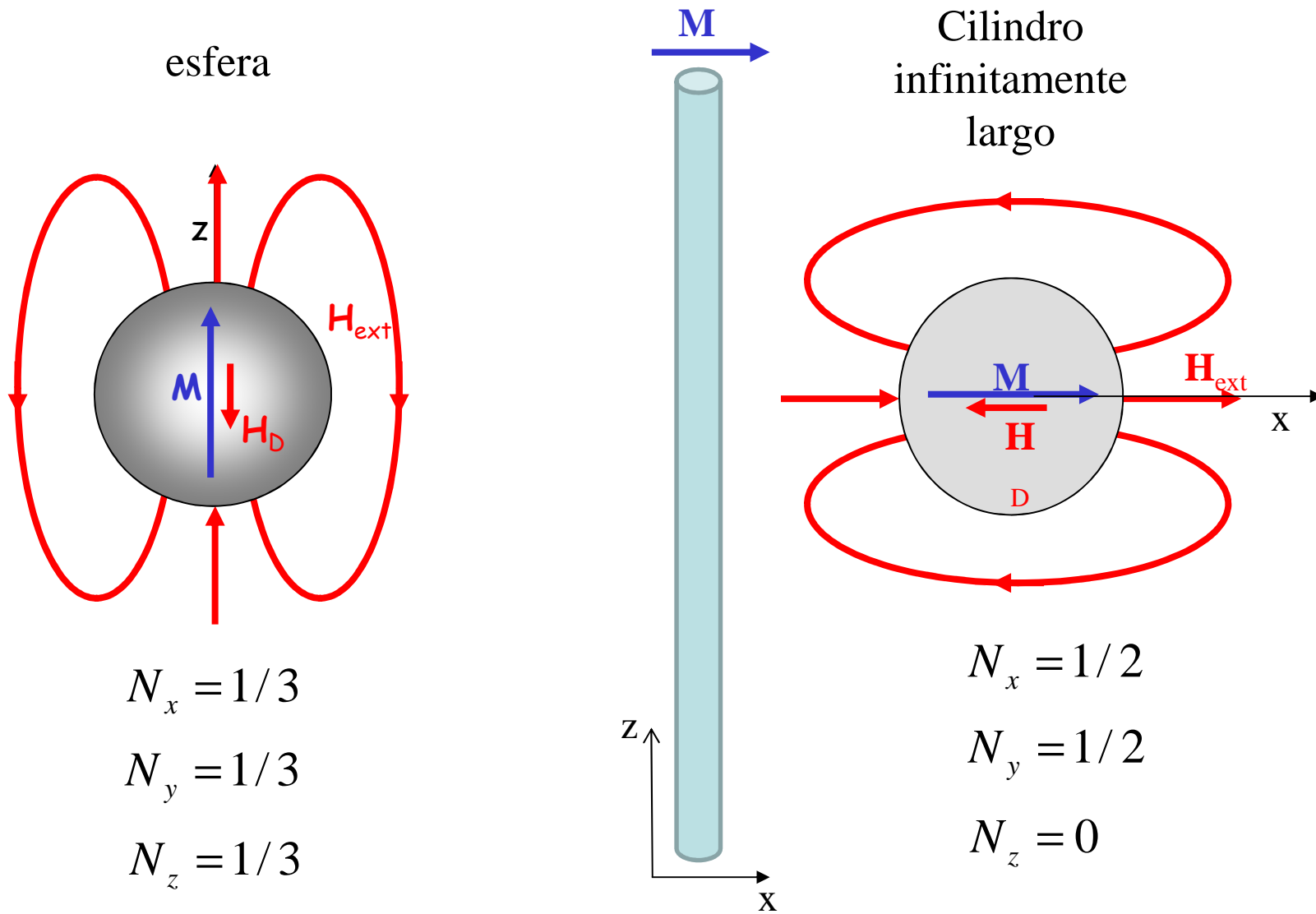
$$H_{\text{int}_z} = -N_D M_S$$

$$\vec{M} = M_S \check{x} \Rightarrow N_x = 1/2$$

$$\vec{M} = M_S \check{y} \Rightarrow N_y = 1/2$$

$$\vec{M} = M_S \check{z} \Rightarrow N_z = 0$$

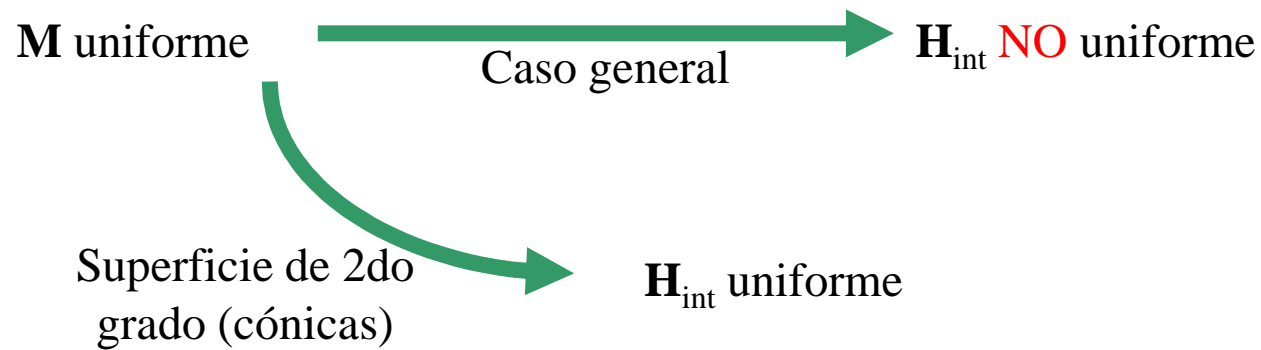
$$N_x + N_y + N_z = 1$$



$$\vec{H}_{Du} = -N\vec{M}_u, \quad u = x, y, z \quad \text{Dirección principal}$$

$$E_M = \frac{\mu_0}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) V$$

Otros cuerpos magnetizados uniformemente



elipsoide

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

\mathbf{M} uniforme

$$\vec{H}_D = -\hat{N}\vec{M}$$

Campo demagnetizante

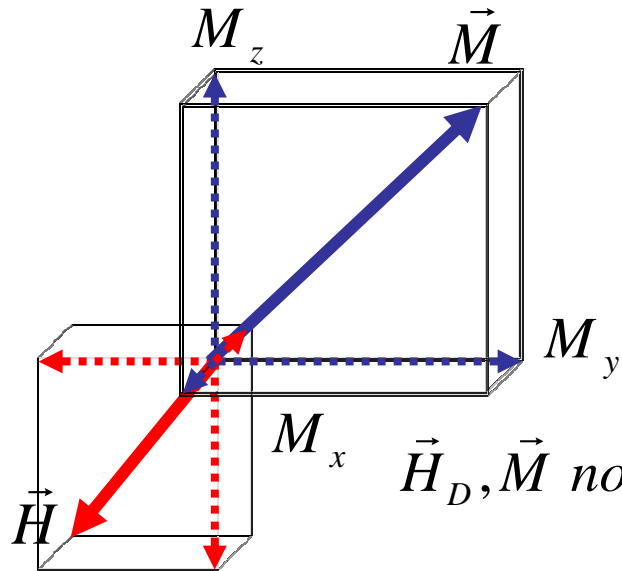
Tensor demagnetizante

Diagonal si los ejes de coordenadas coinciden con los del elipsoide

Traza unitaria
 $N_x + N_y + N_z = 1$

Número si además

$$\vec{M} = \begin{cases} M_s \vec{i} \\ M_s \vec{j} \\ M_s \vec{k} \end{cases}$$



\vec{H}_D, \vec{M} no colineales

$M_i, H_i, unif.$

$$\vec{H}_D = -\hat{N}\vec{M} \quad \Rightarrow$$

$$H_x = -N_x M_x$$

$$H_y = -N_y M_y$$

$$H_z = -N_z M_z$$

$$E_M = -\frac{\mu_0}{2} (M_x H_x + M_y H_y + M_z H_z) V$$

$$E_M = \frac{\mu_0}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) V$$

$$E_M^{min} = 0, \text{ para } M = 0$$

Superficies no cuadráticas

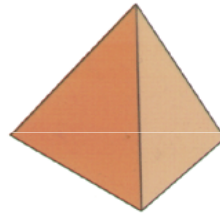
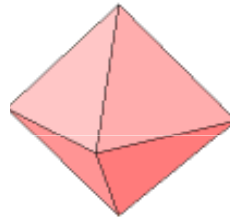
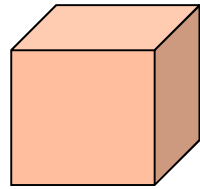
$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

Válido también para cuerpos con superficies no cuadráticas: cubos, prismas, cilindros, octaedros, etc.

(teorema de Brown-Morrish)

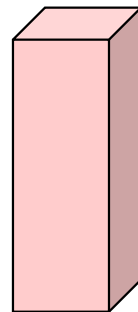
Casos particulares

Cubo,
octaedro,
tetraedro



$$N_x = N_y = N_z = 1/3$$

Prisma
regular,
cilindro

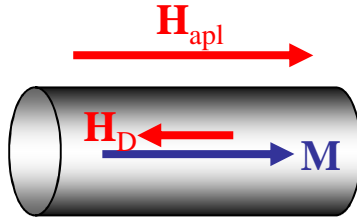


$$N_x = N_y \neq N_z$$

Caso límite $z \gg x, y$

$$N_x = N_y = \frac{1}{2}; \quad N_z = 0$$

campo efectivo



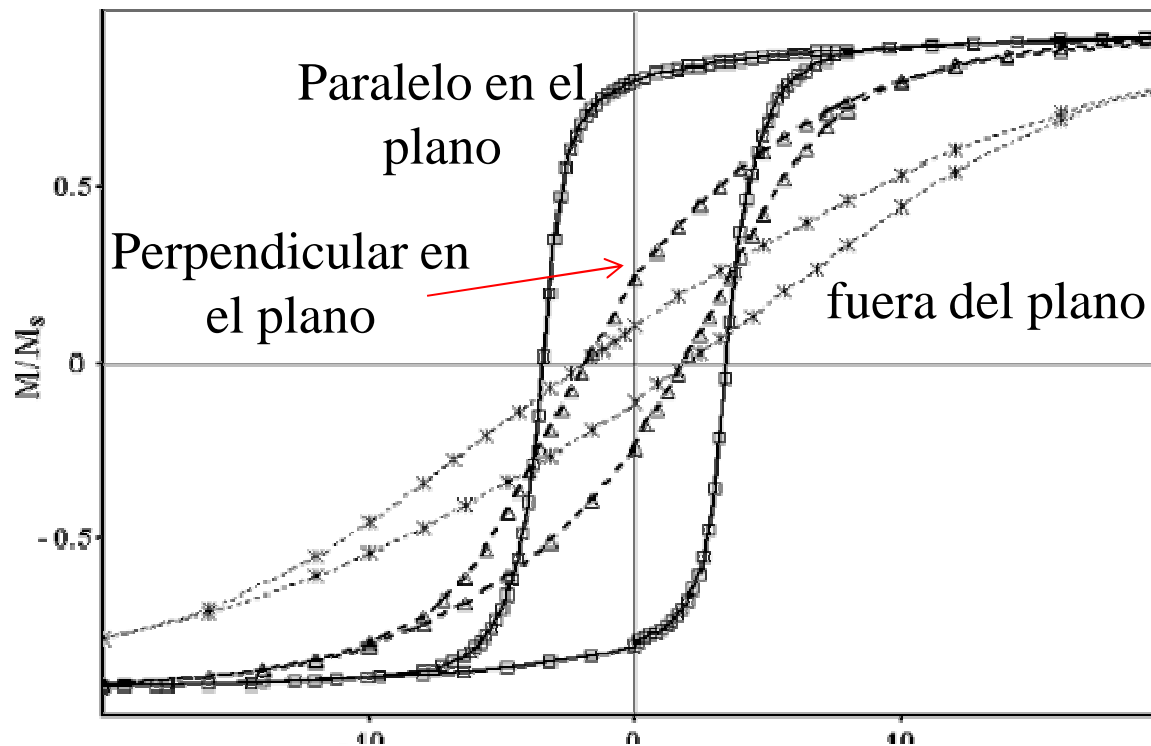
$$H_D = -NM$$

$$H_{ef} = H_{apl} + H_D = H_{apl} - NM$$

Cuando se grafica M vs. H debería usarse como abscisa el H_{ef}

$$M \quad vs \quad H_{ef} = H_{apl} - NM$$

Película de maghemita



factores demagnetizantes

Cálculos en prismas

Demagnetizing factors for rectangular ferromagnetic prisms

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TABLE I. The demagnetizing factor, D_z^s , of a prolate spheroid and the magnetometric demagnetizing factor, D_z^p , of a square prism, for an aspect ratio, p .

p	D_z^s	D_z^p
2.0	0.17356	0.19832
3.0	0.10871	0.14036
4.0	0.075407	0.10845
5.0	0.055821	0.088316
6.0	0.043230	0.074466
7.0	0.034609	0.064363
8.0	0.028421	0.056670
9.0	0.023816	0.050617
10.0	0.020286	0.045731
11.0	0.017515	0.041705

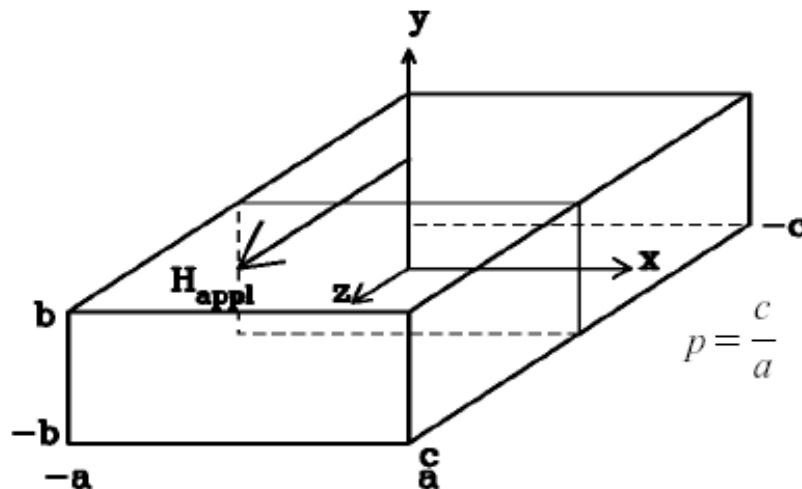


FIG. 1. The coordinate system used in the calculations. Its origin is at the center of the rectangular prism. The field H_{appl} is applied along the z axis.

Factores demagnetizantes– referencias

Fórmulas, tablas y gráficos de factores demagnetizantes, Chen et al. IEEE Trans. Magnetics **27**, 3601-19 (1991)

Campo demagnetizante y medidas magnéticas, J.A. Brug y W.P. Wolf, J.Appl.Phys. **57**, 4685-701 (1985)

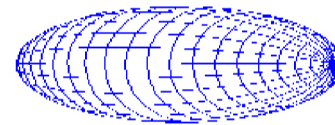
Cálculo de factores demagnetizantes,
<http://www.magpar.net/static/magpar/doc/html/demagcalc.html>

Anisotropía de forma: NP esferoidales

Elipsoide prolado u oblado

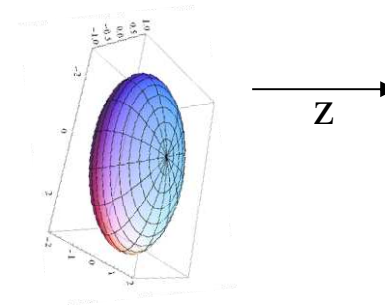
$$a = b < c \Rightarrow N_x = N_y > N_z$$

prolado



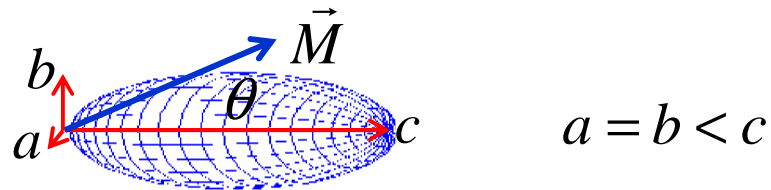
$$a = b > c \Rightarrow N_x = N_y < N_z$$

oblado



Anisotropía de forma: NP elipsoidales

Problema: analizar la energía magnetostática de NP elipsoidales



A partir de la expresión:

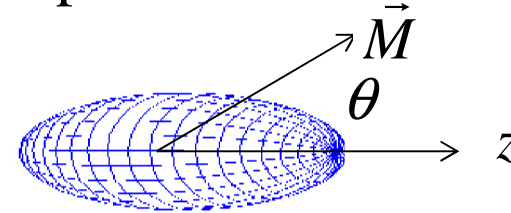
$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

Comparar con la expresión de la energía de anisotropía uniaxial.
Calcular su valor para una NP de magnetita con forma de elipsoide prolado muy alargado, con:

$$M_s \approx 4.5 \times 10^5 \text{ A/m}$$

Anisotropía de forma: NP elipsoidales

$$a = b < c \Rightarrow N_x = N_y > N_z$$



$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) \xrightarrow{N_y=N_x} \frac{\mu_0 V}{2} (N_x (M_x^2 + M_y^2) + N_z M_z^2)$$

$$M_S^2 = M_x^2 + M_y^2 + M_z^2$$

$$E_M = \frac{\mu_0 V}{2} (N_z - N_x) M_z^2 + cte = \frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \cos^2 \theta + cte$$

$$E_M = -\frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \sin^2 \theta + cte = K_{ME} V \sin^2 \theta + cte$$

$$E_M = K_{ME} V \sin^2 \theta$$

$$K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2$$

Ejemplo: elipsoide prolado largo de magnetita con elevada relación de aspecto

$$K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2; \quad N_x \approx 1/2, N_z \approx 0$$

$$M_S \approx 4.5 \times 10^5 \text{ A/m}$$

magnetita

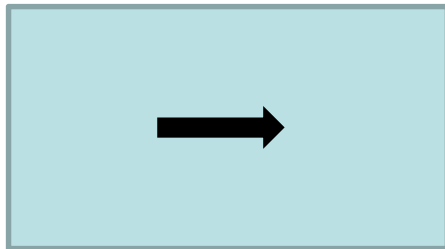
$$K_{ME} = \frac{4\pi \times 10^{-7}}{2} \frac{1}{2} (4.5 \times 10^5)^2 \text{ J/m}^3 \approx 6.4 \times 10^4 \text{ J/m}^3$$

anisotropía magnetocristalina

$$K_{MC} \approx 10^4 \text{ J/m}^3$$

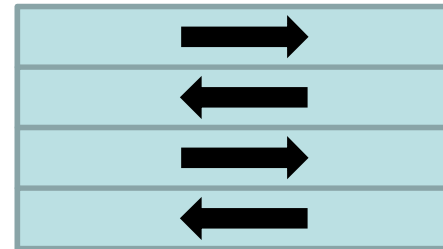
Energía magnetostática - Origen de los dominios

1 dominio



$$E_M(1) = \frac{NV}{2} \mu_0 M_S^2$$

n dominios

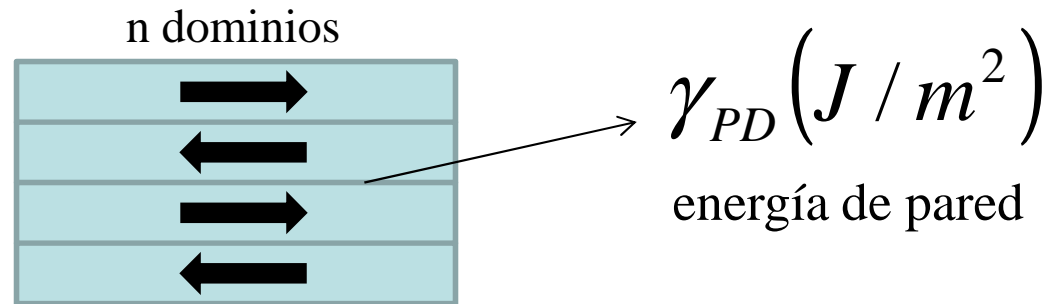


$$\frac{E_M(n)}{E_M(1)} = \frac{E_M(n)}{E_M^{\text{mono}}} \approx \frac{1}{n}$$

$$E_M(n) \approx \frac{E_M^{\text{mono}}}{n} \quad \text{Tendencia aproximada}$$

$$E_M(n) = C(n) \mu_0 M_S^2$$

Energía de pared de dominios

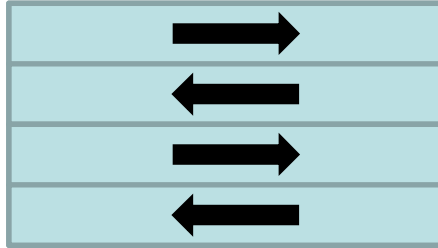


$$E_{PD} \approx S_{PD}(n) \gamma_{PD} \left(J / m^2 \right)$$

↑
superficie de las
paredes

Número de dominios en equilibrio

n dominios



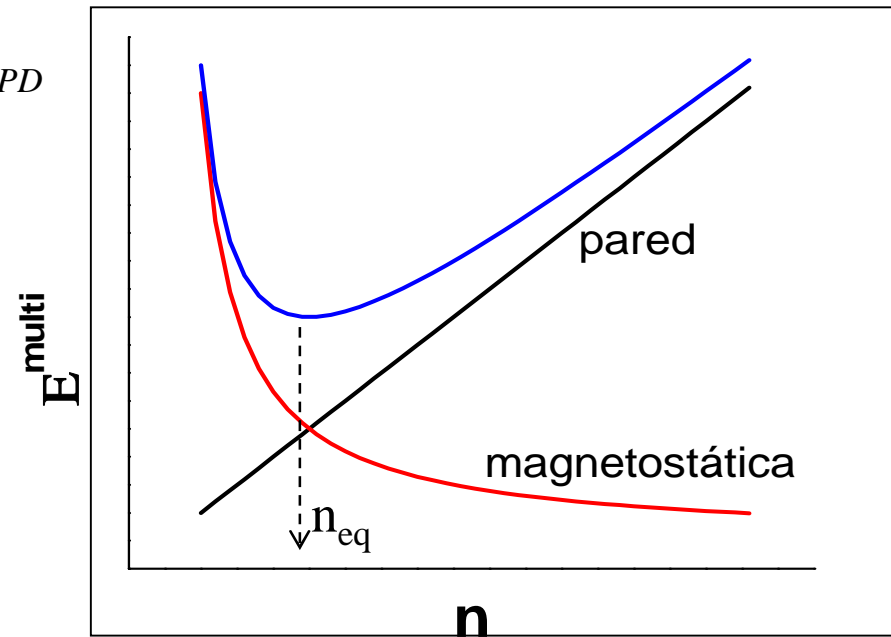
Energía de pared por unidad de área

$$E^{multi} \approx \frac{E_M^{mono}}{n} + S_{PD}(n)\gamma_{PD}$$

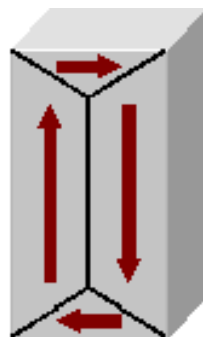
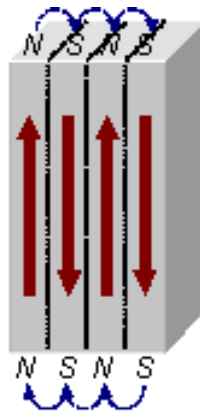
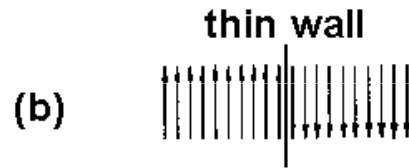
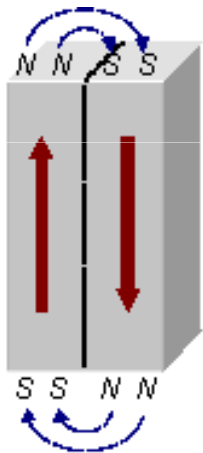
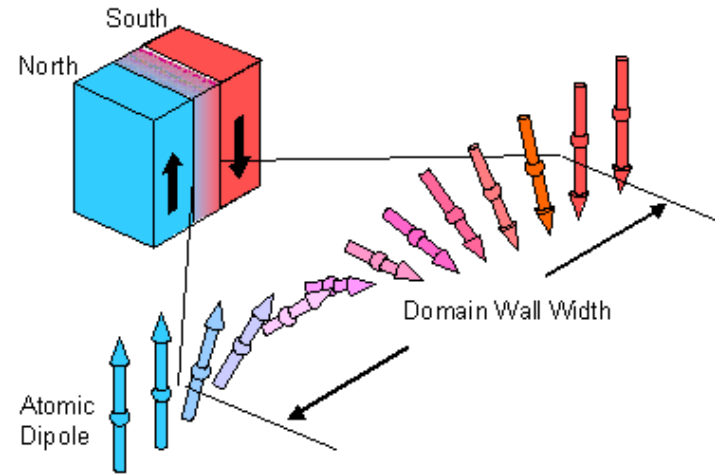
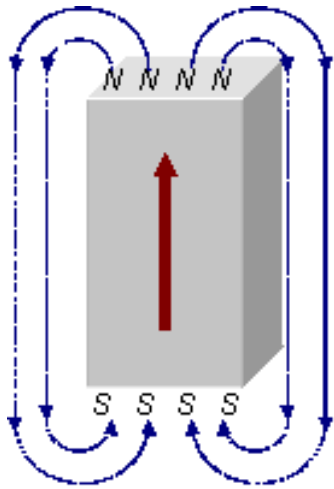
E magnetostática decrece

E pared dominios crece

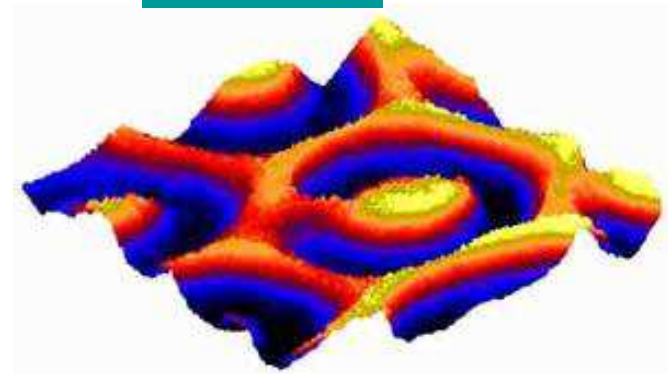
Nro dominios en equilibrio



Dominios y paredes de dominio

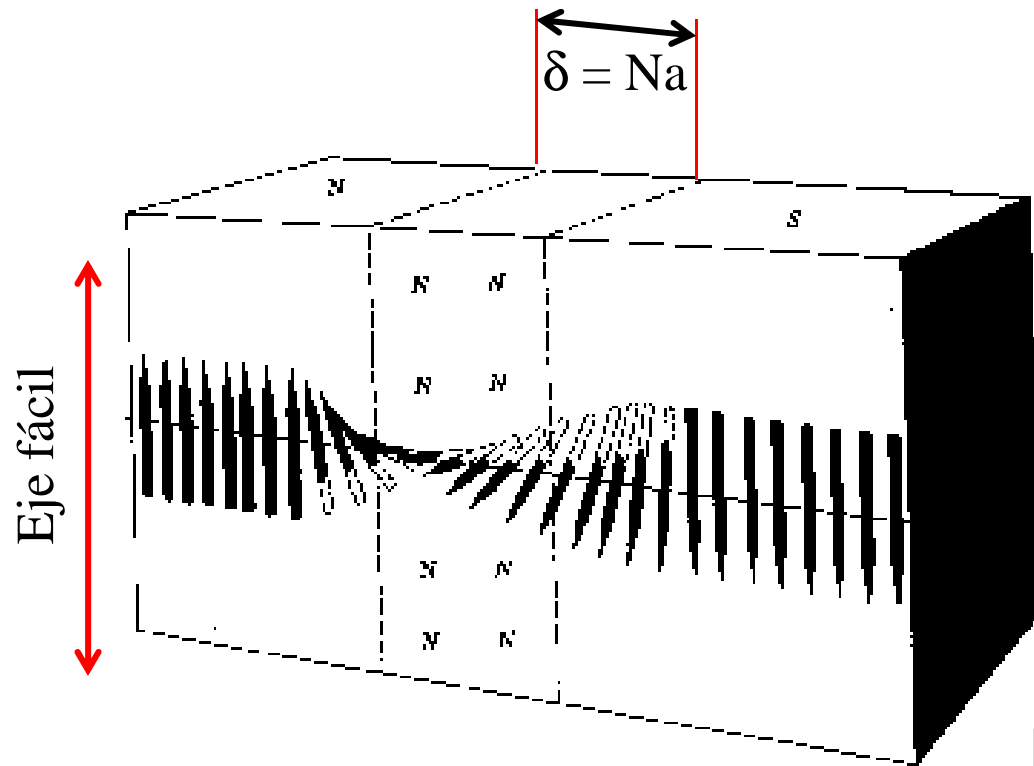


MFM

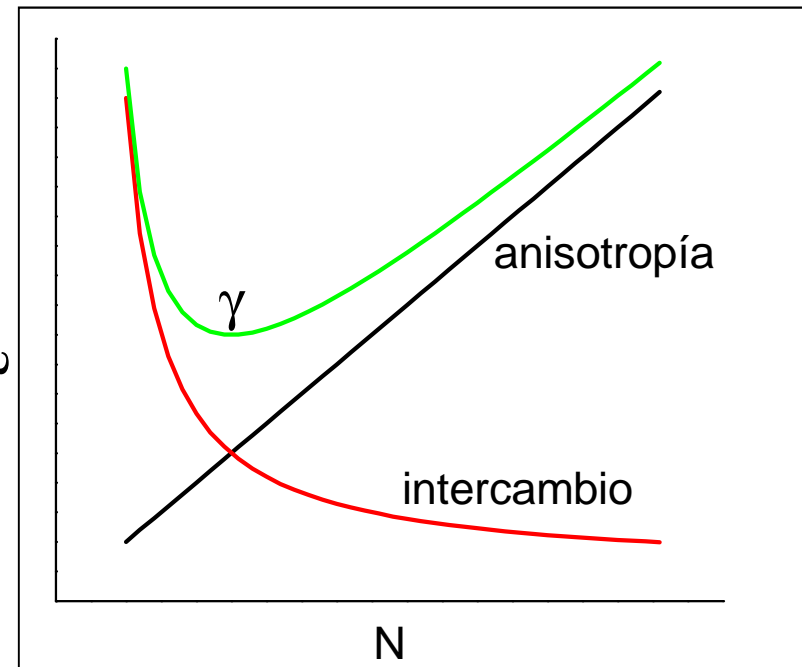


Pseudo-3d MFM image of a $(\text{YSmLaCa})_3 (\text{FeGe})_5 \text{O}_{12}$ magnetic thin film garnet, $4.5 \times 4.5 \mu\text{m}^2$, domain walls appear dark;

Pared de Bloch de 180°



$$\gamma = \Delta\epsilon_K + \Delta\epsilon_J$$



Optimización energía por unidad de área de pared

$$\gamma = \Delta\varepsilon_K + \Delta\varepsilon_J$$

Ancho de la pared

$$\delta_{eq} = N_{eq} a = \pi \sqrt{\frac{2Js^2}{Ka}} = \pi \sqrt{\frac{2A}{K}}$$

$$A \approx Js^2 / a$$

Red cúbica
simple,
Cte de stiffness

$$\gamma_{eq} = 2\pi\sqrt{2KA}$$

$$10^{-12} J / m \leq A \leq 10^{-11} J / m$$

$$10^3 J / m^3 \leq K \leq 10^6 J / m^3$$

$$A = 10^{-11} J / m$$

$$K = 10^3 J / m^3 \Rightarrow \delta_{eq} = 444 \text{ nm}$$

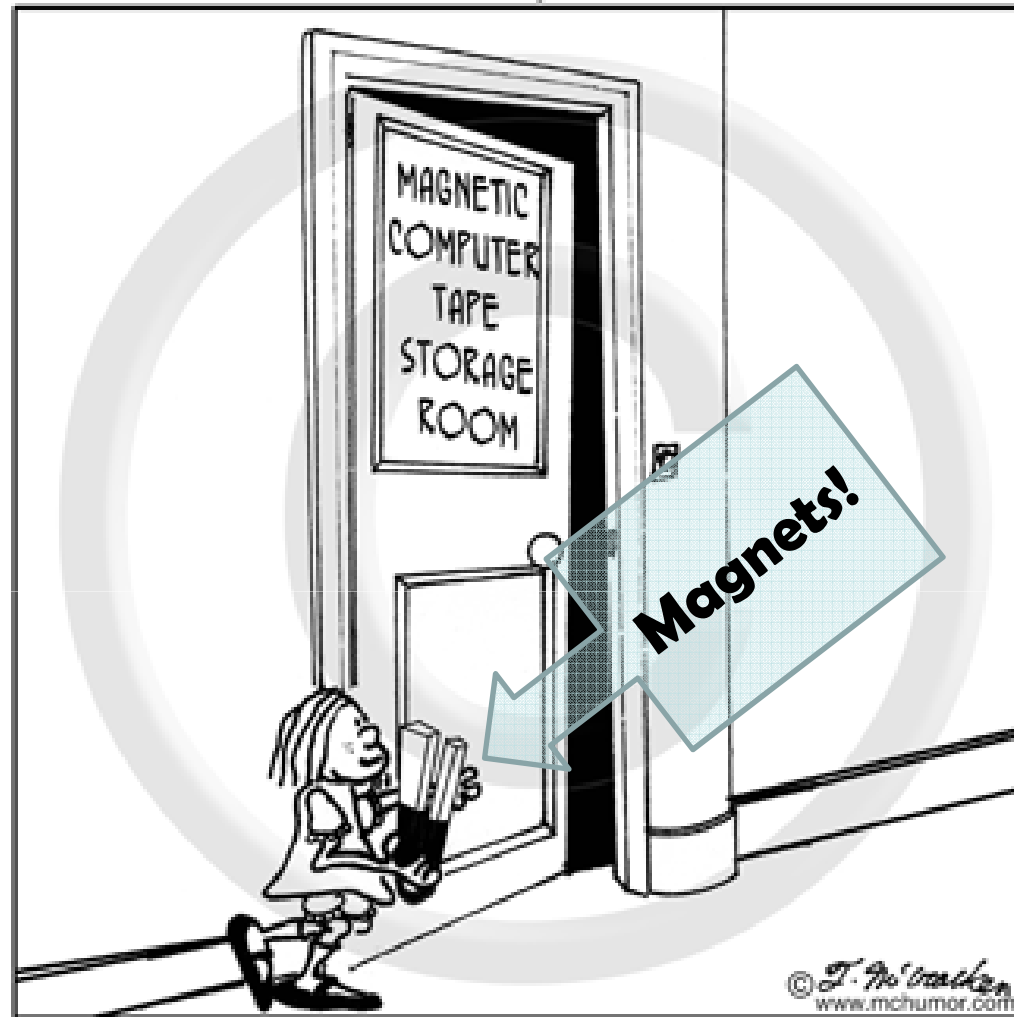
$$K = 10^5 J / m^3 \Rightarrow \delta_{eq} = 44.4 \text{ nm}$$

$$d_c = \frac{72}{\mu_0 M_S^2} \sqrt{AK}$$

$$A = nJs^2 / a \quad \begin{cases} n = 1 & (sc) \\ n = 2 & (bcc) \\ n = 4 & (fcc) \end{cases}$$

$$K = K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2$$

$$d_c = \frac{72}{\sqrt{\mu_0} M_S} \sqrt{A(N_x - N_z) / 2}$$



21st Century Childhood Pranks.