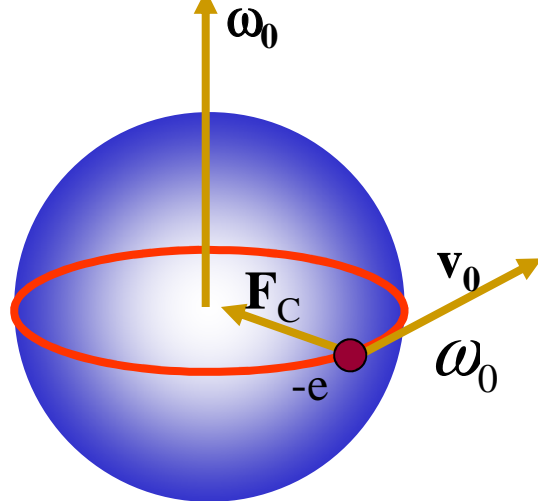


# Diamagnetismo y paramagnetismo

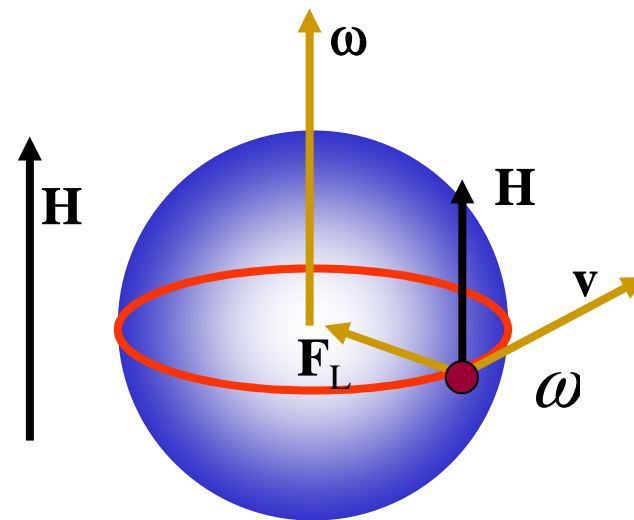
# Diamagnetismo

En ausencia de campo magnético



$$F_C = F_e = eE(r)$$

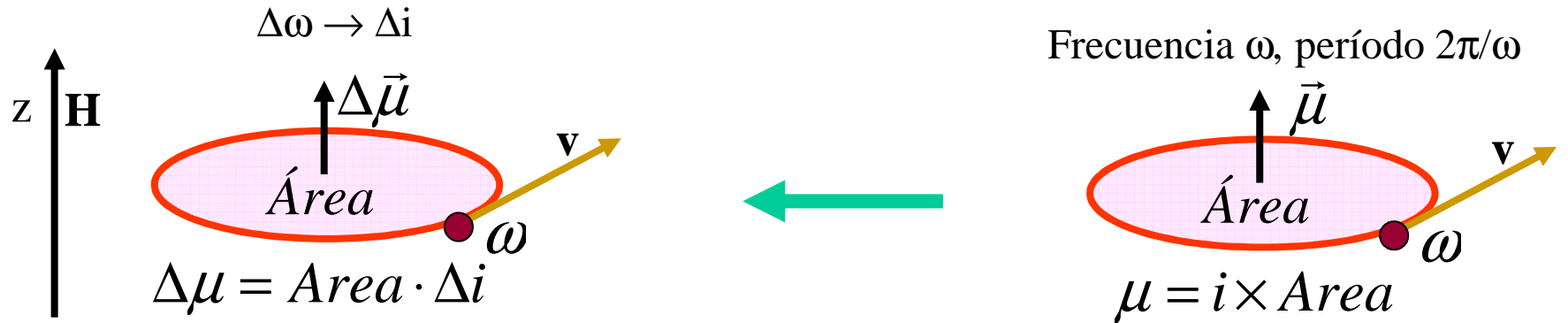
En presencia de campo magnético



$$\vec{F}_L = -e\vec{v} \times \vec{B} \Rightarrow F_L = -\mu_0 evH$$

$$\vec{F}_C = \vec{F}_e + \vec{F}_L$$

# Diamagnetismo



$$\Delta\chi = \frac{\Delta M}{H} = N \frac{\Delta\mu}{H}$$

Para un ión con Z electrones

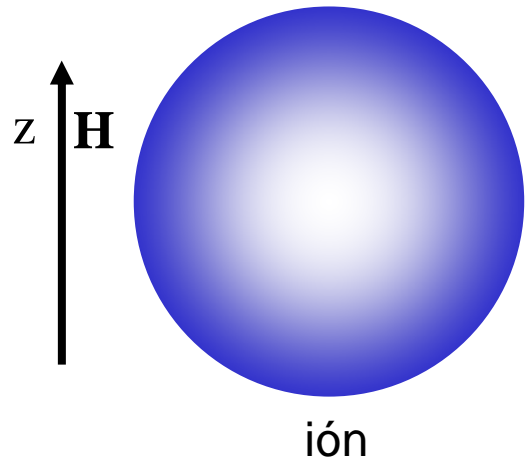
$$\chi_{dia} = -\frac{N\mu_0 e^2}{6m} \sum_{i=1}^z \overline{r_i^2}$$

Diamagnetismo de Langevin

## Susceptibilidades magnéticas de algunos materiales

Material	T (K)	P(atm)	$\chi_m$ (m <sup>3</sup> /kg)	$\chi_v$
agua	293	1	$-9.05 \times 10^{-9}$	$-9.04 \times 10^{-6}$
bismuto	293	1	$-1.70 \times 10^{-8}$	$-1.66 \times 10^{-4}$
diamante	RT	1	$-6.2 \times 10^{-9}$	$-2.2 \times 10^{-5}$
aire	293	1		$3.6 \times 10^{-7}$
N <sub>2</sub>	293	0.78	$-5.56 \times 10^{-9}$	$-5.06 \times 10^{-9}$
O <sub>2</sub>	293	0.21	$1.34 \times 10^{-6}$	$3.73 \times 10^{-7}$

# Diamagnetismo y Paramagnetismo



$$\chi = -\frac{e\hbar}{2m} Ng \frac{\partial \langle J_z \rangle}{\partial H} - \frac{\mu_0 e^2 N}{6m} \sum_{i=1}^Z \langle r^2 \rangle_i$$

momento angular
paramagnetismo
diamagnetismo

Poco dependiente de T

Magnetón  
de Bohr

$$\mu_B = -\frac{e\hbar}{2m} \quad \longrightarrow \quad \chi_{para} = -Ng\mu_B \frac{\partial \langle \vec{J}_z \rangle}{\partial H}$$

# Paramagnetismo

## Momento permanente

$$\langle \mu_z \rangle = -\mu_B g \sum_{i=1}^Z \langle j_z \rangle = -\mu_B g \langle J_z \rangle \quad \longrightarrow \quad \vec{\mu} = -\mu_B g \vec{J}$$

$$\vec{\mu} = -g \mu_B \vec{J}$$

Factor de Landé

$$\vec{J} = \vec{L} \Rightarrow g = 1$$

$$\vec{J} = \vec{S} \Rightarrow g = 2$$

$$\vec{J} = \vec{L} + \vec{S} \Leftrightarrow g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

# Paramagnetismo

$$\vec{\mu} = -g\mu_B\vec{J}$$

$$\mu^2 \Leftrightarrow J^2$$

$$|\vec{\mu}| = g \left[ \langle J^2 \rangle \right]^{1/2} \mu_B$$

Medidas de susceptibilidad

$$|\vec{\mu}| = g [J(J+1)]^{1/2} \mu_B$$

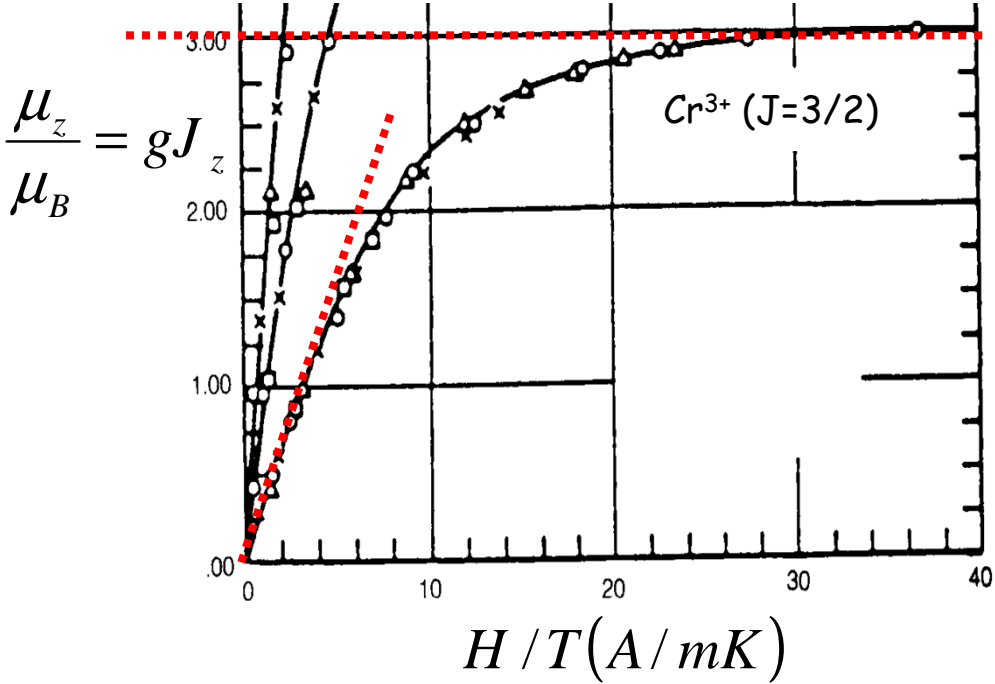
$$\mu_z \Leftrightarrow J_z$$

Medidas de magnetización de saturación

$$\mu_z = gJ_z\mu_B$$

$$\mu_z^{m\acute{a}x} = gJ\mu_B = \mu$$

$$|\mu| = \langle \mu^2 \rangle^{1/2}$$



# Paramagnetismo

“Quenching” del momento angular en iones 3d

$$\vec{J} = \vec{L} + \vec{S} \approx \vec{S}$$

$\vec{L}$  no es una constante del movimiento, no conmuta con el Hamiltoniano.

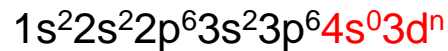
Su valor medio promedia a cero

$$\langle \vec{L} \rangle \approx 0$$



# Paramagnetismo en iones 3d

	V <sup>+2</sup> Cr <sup>+3</sup>	Cr <sup>+2</sup> Mn <sup>+3</sup>	Mn <sup>+2</sup> Fe <sup>+3</sup>
3d	↑ ↑ ↑	↑ ↑ ↑ ↑	↑ ↑ ↑ ↑ ↑
4s			
3p	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓
3s	↑↓	↑↓	↑↓
2p	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓	↑↓ ↑↓ ↑↓
2s	↑↓	↑↓	↑↓
1s	↑↓	↑↓	↑↓



Mn<sup>+2</sup> Fe<sup>+3</sup>

$$|\vec{\mu}| = g [J(J+1)]^{1/2} \mu_B$$

$$J \approx S = 2.5$$

$$\mu = \mu_z^{max} = gJ\mu_B$$

$$J \approx 2.5$$

$$g \approx 2$$

$$|\vec{\mu}| \approx 2 \times (2.5 \times 3.5)^{1/2} \mu_B$$

$$|\vec{\mu}| \approx 5.92 \mu_B$$

$$\mu_z \approx 2 \times 2.5 \mu_B$$

$$\mu_z \approx 5 \mu_B$$

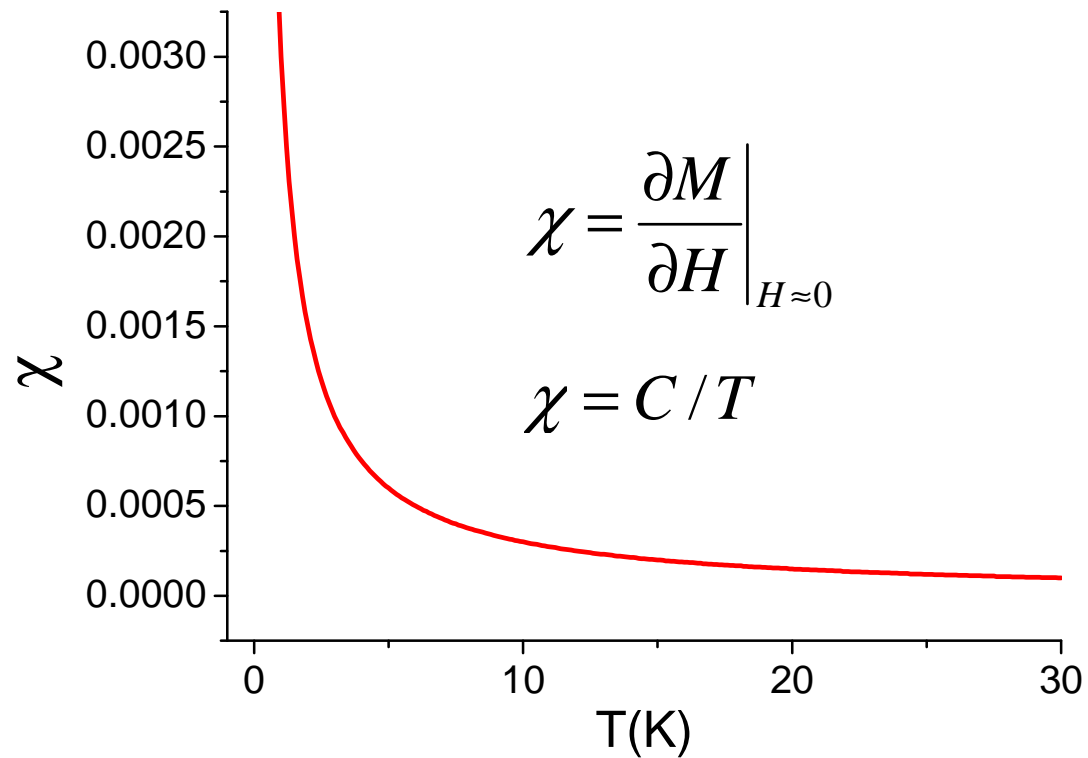
# Paramagnetismo en iones 3d

Momento magnético en magnetones de Bohr

ión	Configuración	g[j(j+1)] <sup>0.5</sup> Calc.	g[s(s+1)] <sup>0.5</sup> Calc.	medido
Ti <sup>3+</sup> , V <sup>4+</sup>	3d <sup>1</sup>	1.55	<b>1.73</b>	<b>1.8</b>
V <sup>3+</sup>	3d <sup>2</sup>	1.63	<b>2.83</b>	<b>2.8</b>
Cr <sup>3+</sup> , V <sup>3+</sup>	3d <sup>3</sup>	0.77	<b>3.87</b>	<b>3.8</b>
Mn <sup>3+</sup> , Cr <sup>3+</sup>	3d <sup>4</sup>	0	<b>4.90</b>	<b>4.9</b>
Fe <sup>3+</sup> , Mn <sup>2+</sup>	3d <sup>5</sup>	5.92	<b>5.92</b>	<b>5.9</b>
Fe <sup>2+</sup>	3d <sup>6</sup>	6.70	4.90	<b>5.4</b>
Co <sup>2+</sup>	3d <sup>7</sup>	6.63	3.87	<b>4.8</b>
Ni <sup>2+</sup>	3d <sup>8</sup>	5.59	2.83	<b>3.2</b>
Cu <sup>2+</sup>	3d <sup>9</sup>	3.55	1.73	<b>1.9</b>

# Paramagnetismo: experimentos

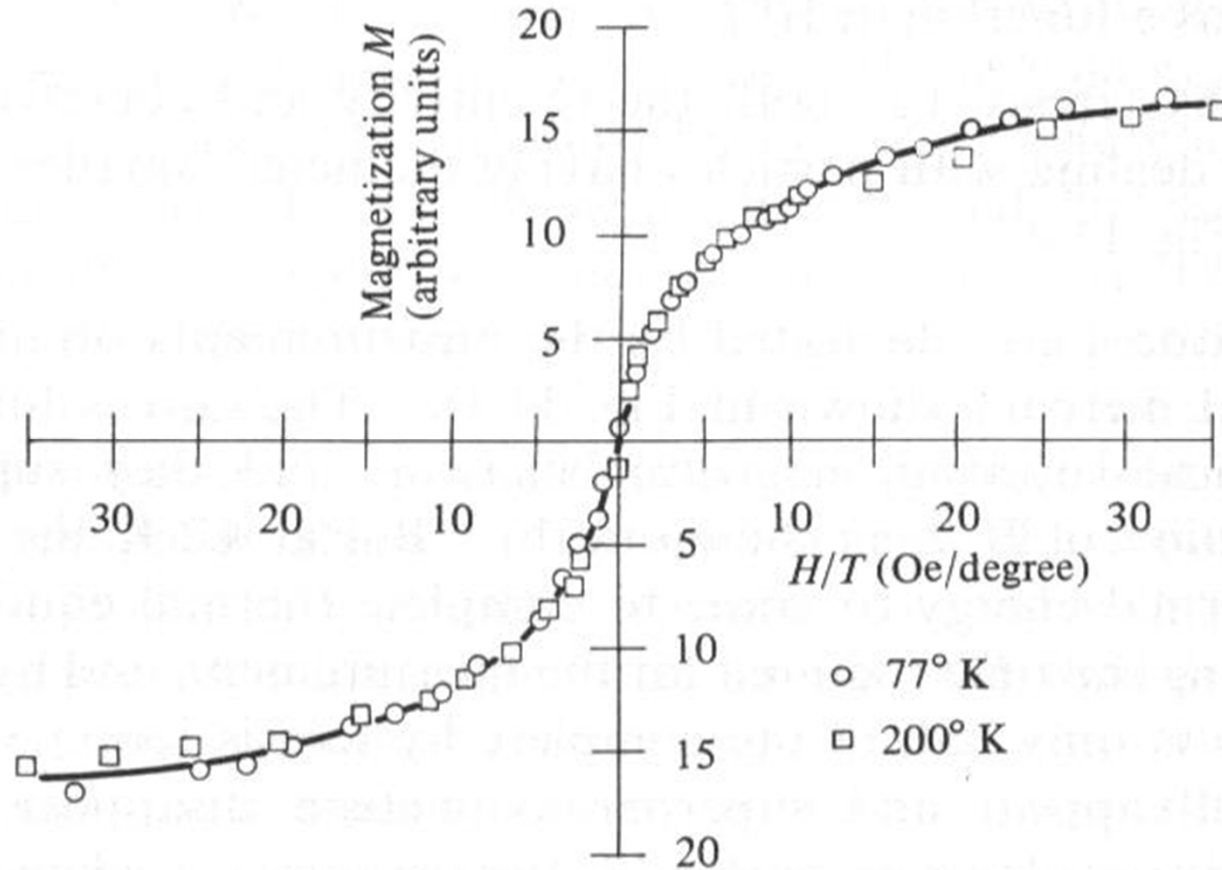
Dependencia de  $M$  con  $T$  y con  $H$



Susceptibilidad de un paramagneto bajo  
pequeños campos ( $H = \text{cte}$ )

# Paramagnetismo: experimentos

Dependencia de  $M$  con  $H$  y con  $T$



Función universal de  $H/T$

## Dependencia de $\mathbf{M}$ con $\mathbf{H}$ y con $T$ . Modelo estadístico

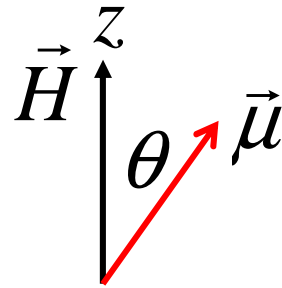
Paramagneto (Curie):

Ausencia de interacciones entre  
espines

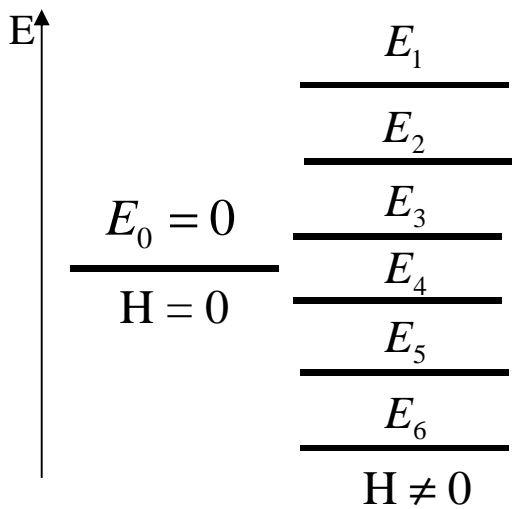
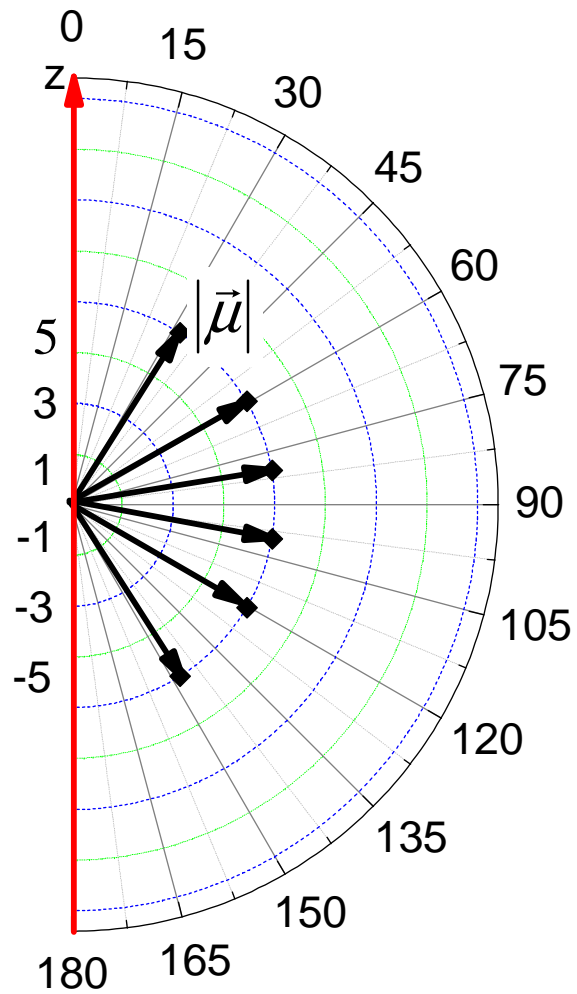
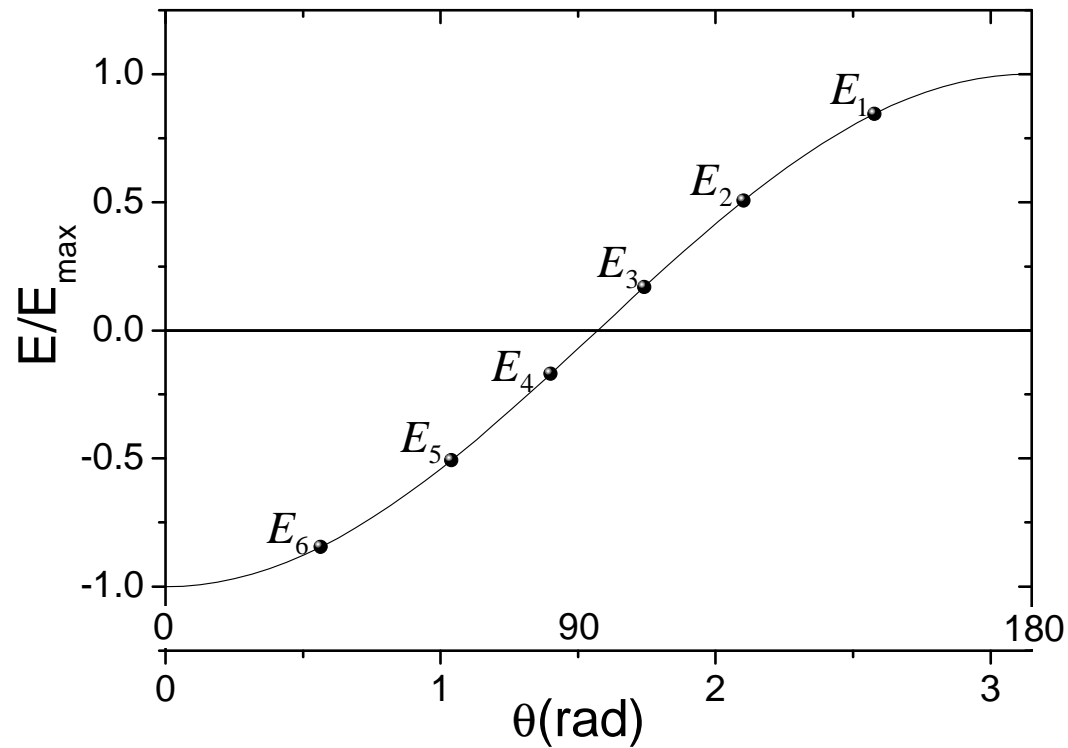
Momento permanente localizado

Ejemplo:  $J = 3/2$

$$\vec{\mu} = -g\mu_B\vec{J}$$



$$E_z = -\vec{\mu} \cdot \vec{B} = -\mu_0 \vec{\mu} \cdot \vec{H} = -\mu_0 \mu_z H = -\mu_0 |\vec{\mu}| H \cos \theta$$

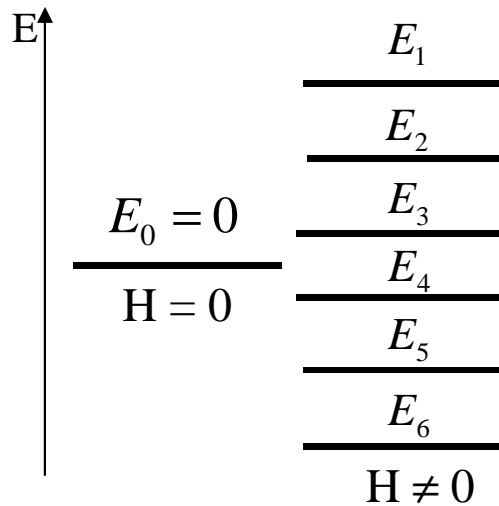


En equilibrio la probabilidad de ocupación de los subniveles de energía es proporcional a los factores de Boltzmann:

$$e^{-E_z/kT} = e^{\mu_0\mu_z H/kT} = e^x \quad x = \mu_0\mu H/kT$$

probabilidad

$$p(\mu_z) = \frac{e^{\mu_0\mu_z H/kT}}{\sum e^{\mu_0\mu_z H/kT}}$$





Se promedia  $\mu_z$  usando las probabilidades  $p(\mu_z)$

solución

$$\frac{\langle \mu_z \rangle}{\mu} = B_J(x) \quad x = \mu_0 \mu H / kT$$

Función de Brillouin

$$B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)$$

$$M = N \langle \mu_z \rangle = N \mu B_J(x)$$

$$\leftarrow M = M(H/T)$$

Número de momentos por unidad de volumen

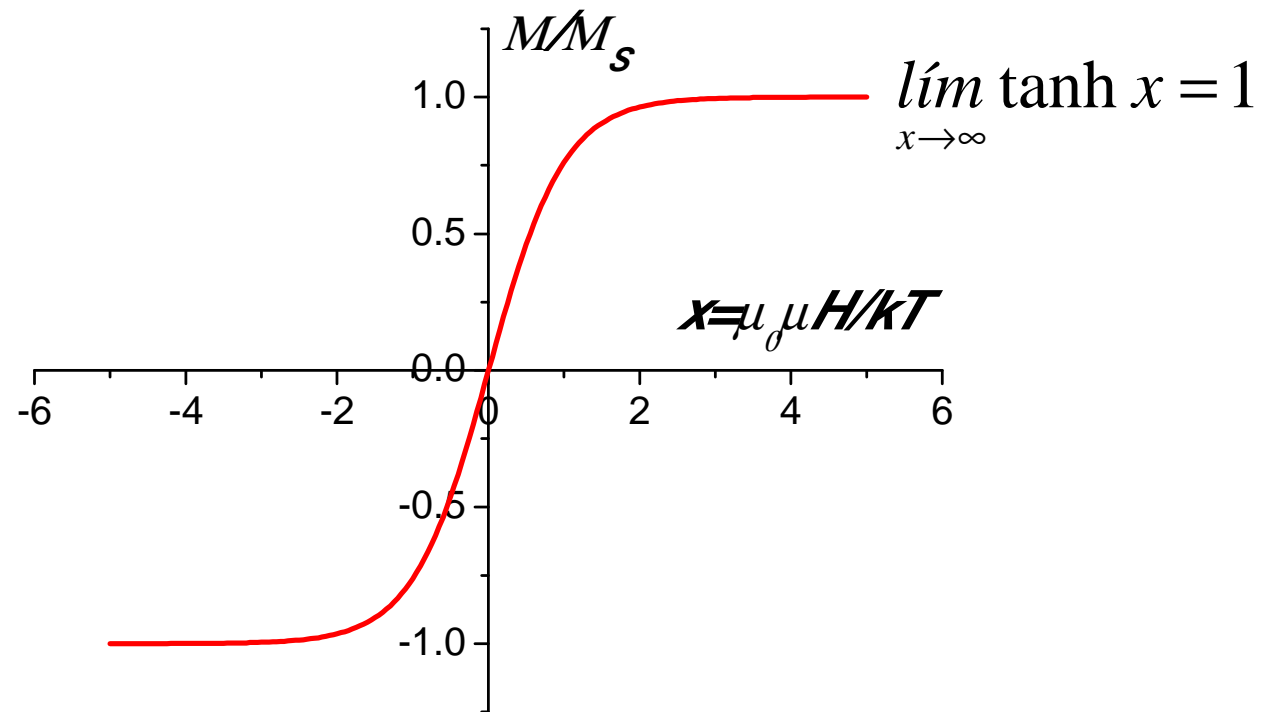
$$J = 1/2$$

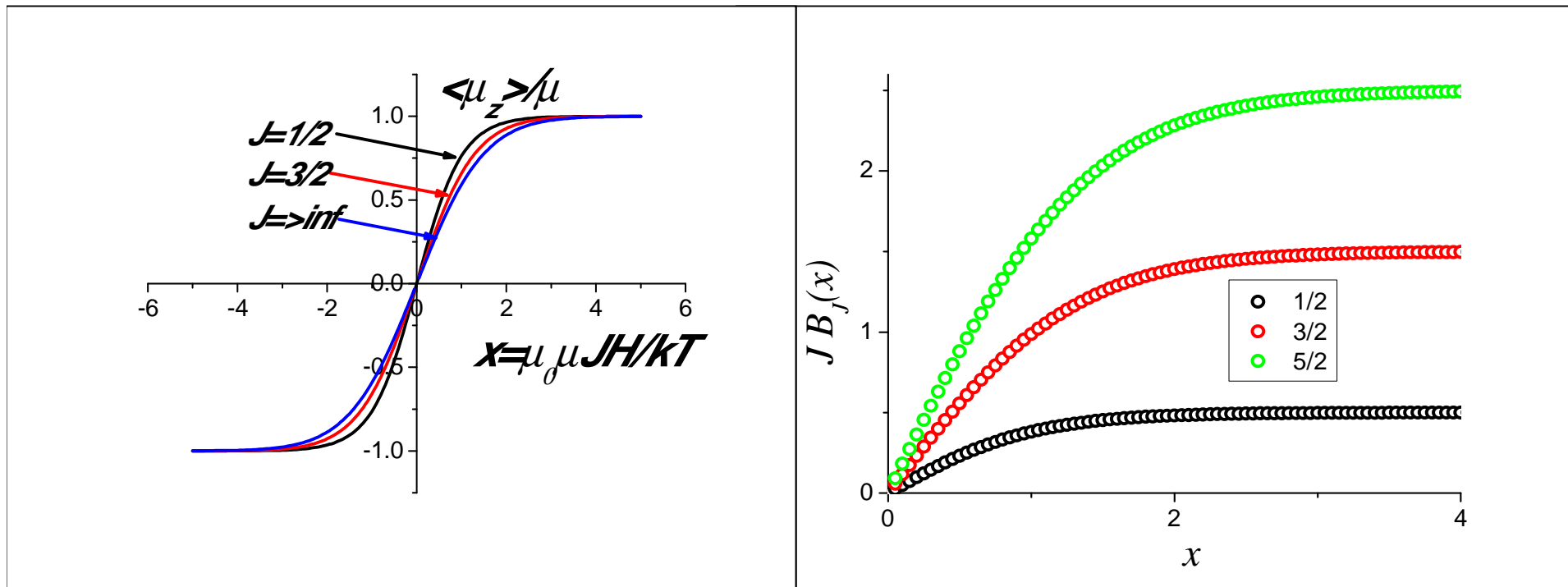
$$g = 2$$

$$B_J(x) = \tanh x$$

$$M(x) = N\mu \tanh x = N\mu_B \tanh x$$

$$M_S(x) = N\mu$$

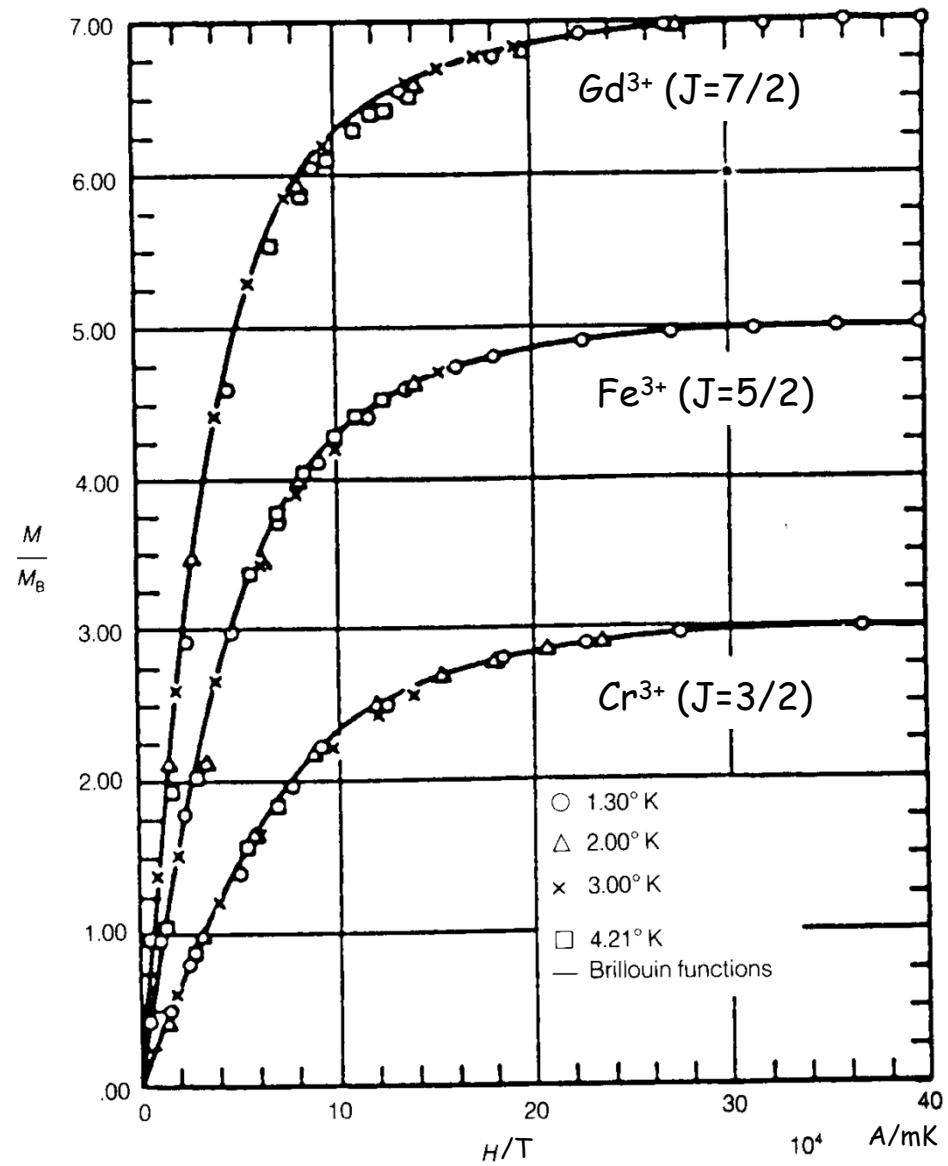




La función “satura” para  $x \geq 4$

Sales  
paramagnéticas  
de:

$\text{Cr}^{3+}$  ( $J=3/2$ )  $\text{Fe}^{3+}$   
( $J=5/2$ )  $\text{Gd}^{3+}$   
( $J=7/2$ )



Buen acuerdo de la  
teoría con los  
resultados  
experimentales

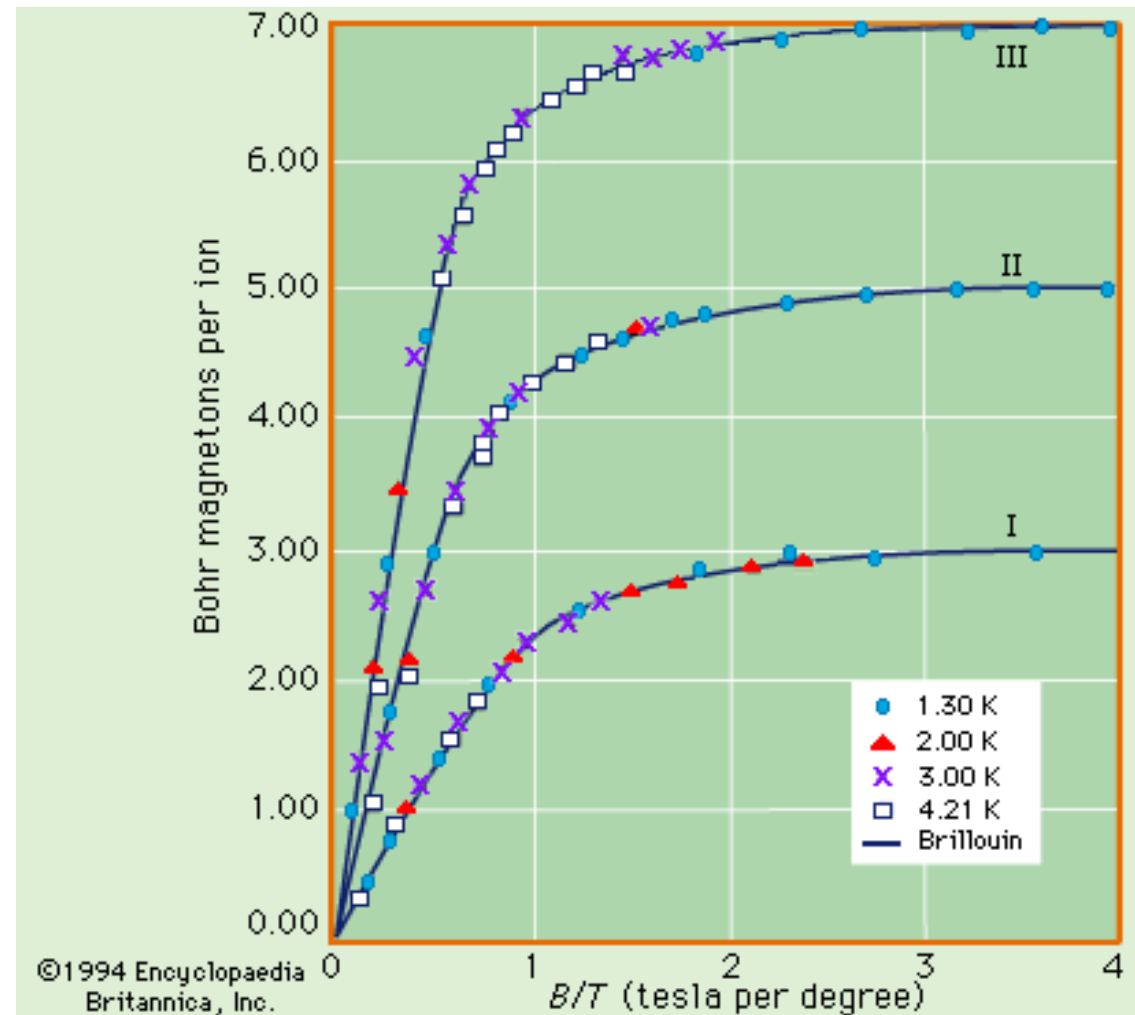
## Comportamientos límites

### Saturación

$$x \rightarrow \infty \quad B_J(x) \rightarrow 1$$

$$M = M_S = N\mu$$

De la medida de  $M_S$  se obtiene el valor de la proyección máxima del momento en la dirección del campo



Campo necesario para saturar un conjunto de iones paramagnéticos de un metal de transición con  $J = 1/2$

$$x = \frac{\mu_0 \mu H}{kT} \quad \mu = g \mu_B J$$

$$\left. \begin{array}{l} \mu_0 = 4\pi \times 10^{-7} \text{ (SI)} \\ \mu_B = 9.27 \times 10^{-24} \text{ (SI)} \\ k = 1.38 \times 10^{-23} \text{ (SI)} \\ g = 2; J = 1/2 \end{array} \right\} x = 8.44 \times 10^{-6} \frac{H \text{ (A/m)}}{T \text{ (K)}}$$

$$x = 5$$

$$T = 1K \Rightarrow H_s \geq 5.92 \times 10^5 \text{ A/m} \Rightarrow B_s \geq 0.74T$$

$$T = 300K \Rightarrow H_s \geq 1.78 \times 10^8 \text{ A/m} \Rightarrow B_s \geq 222T \quad \text{!!!!}$$

## Comportamientos límite

### Medidas de susceptibilidad y magnetización inicial

$$x \rightarrow 0 \ (x \ll 1)$$

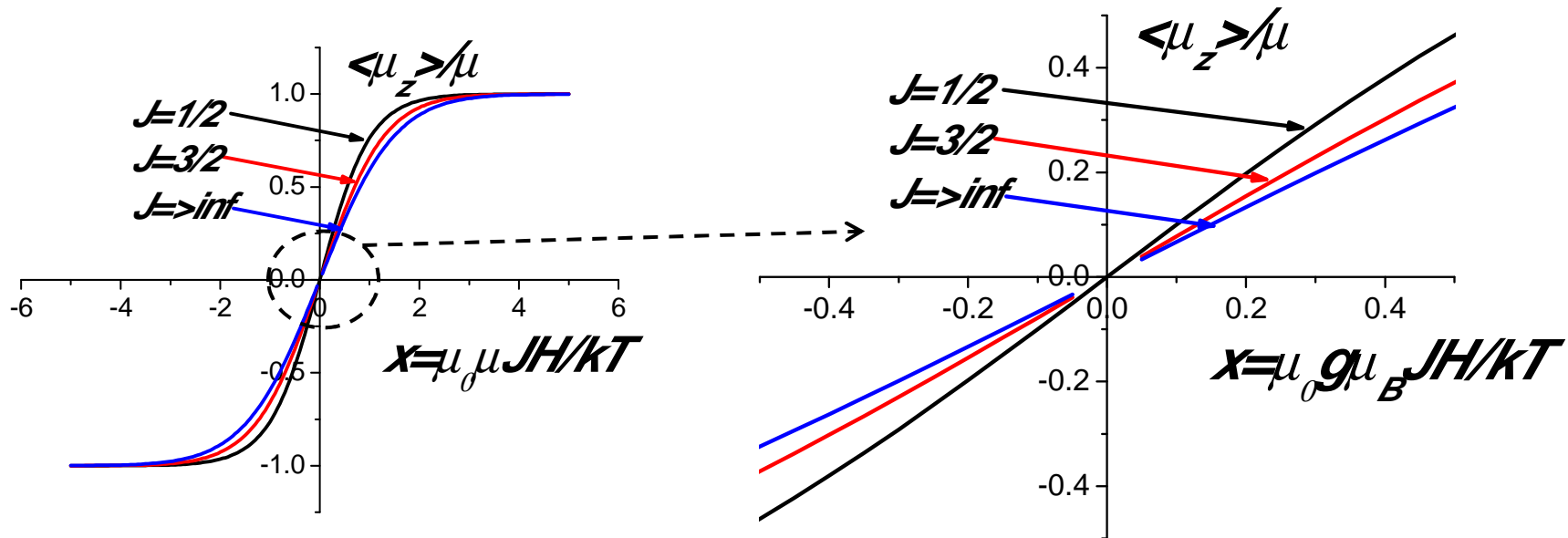
$$B_J(x) \approx \frac{J+1}{3J} x + \cancel{O(x^3)}$$

$$x = \frac{\mu_0 \mu H}{kT}$$

$$\langle \mu_z \rangle = \mu B_J(x) \approx \mu \frac{(J+1)}{3J} x$$

$$\langle \mu_z \rangle \approx (g\mu_B J) \frac{(J+1)}{3J} \left( \frac{\mu_0 g \mu_B J H}{kT} \right) = \frac{\mu_0 \mu^2 H}{3kT}$$

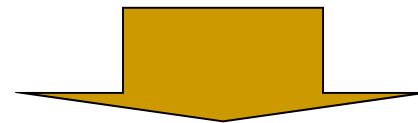
$$M = N \langle \mu_z \rangle = \frac{N \mu_0 |\mu|^2 H}{3kT}; \quad |\mu|^2 = \langle \mu^2 \rangle$$



Dependencia lineal con H,

$$x \leq 0.2$$

$$T = 300 \text{ K}$$

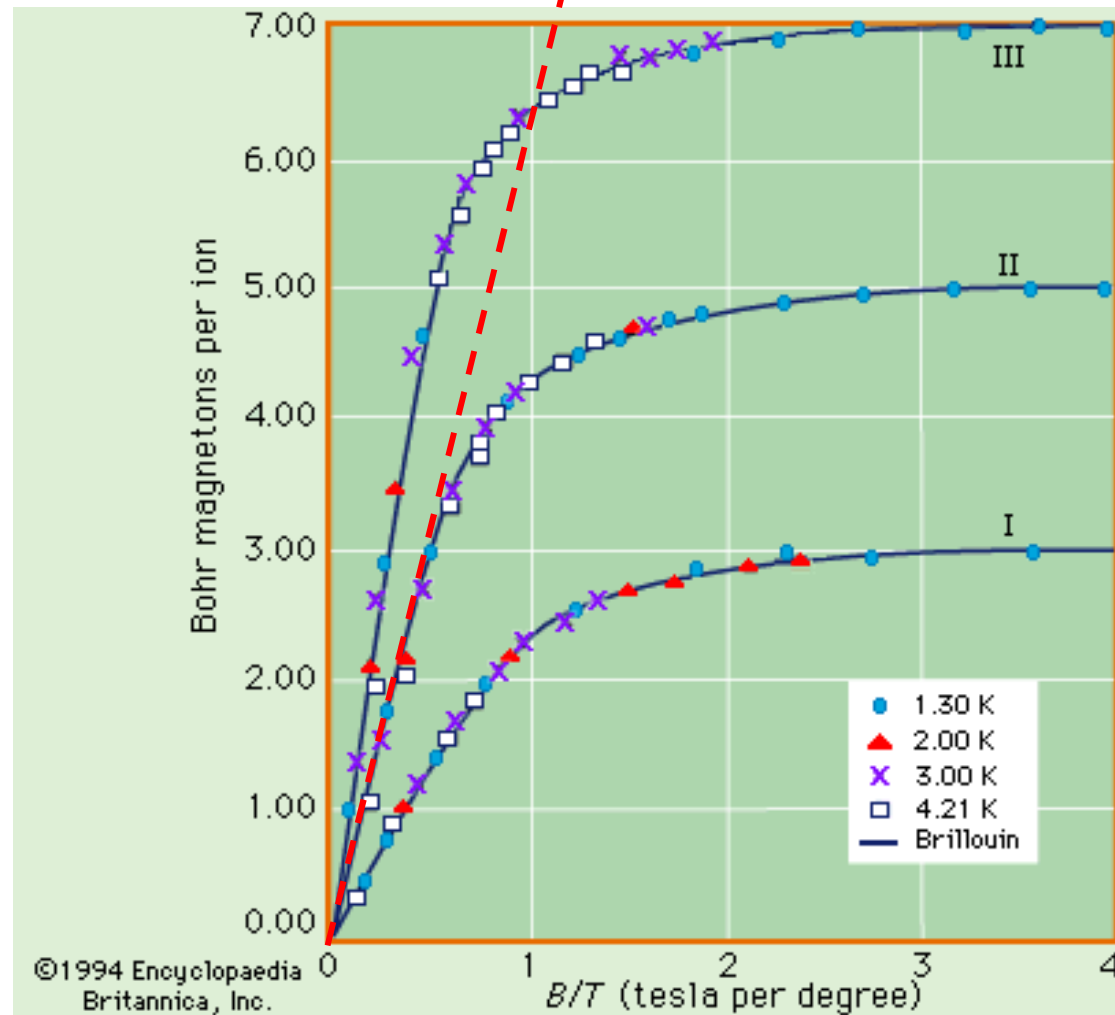


$$H_i \leq 2.4 \times 10^4 \text{ A/m} \approx 300 \text{ Oe} \Rightarrow B_i \leq 30 \text{ mT}$$



$$\chi_{inic} = \frac{M_{inic}}{H} \approx \frac{N\mu_0 \langle \mu^2 \rangle}{3kT}$$

Información sobre  
N,  $\mu$ , g, J, T



Susceptibilidad inicial

$$\mu^2 = g^2 \mu_B^2 J(J+1)$$

$$\chi = \frac{M}{H} \approx \frac{N\mu_0 \mu^2}{3kT} = \frac{C}{T}$$

Constante de Curie

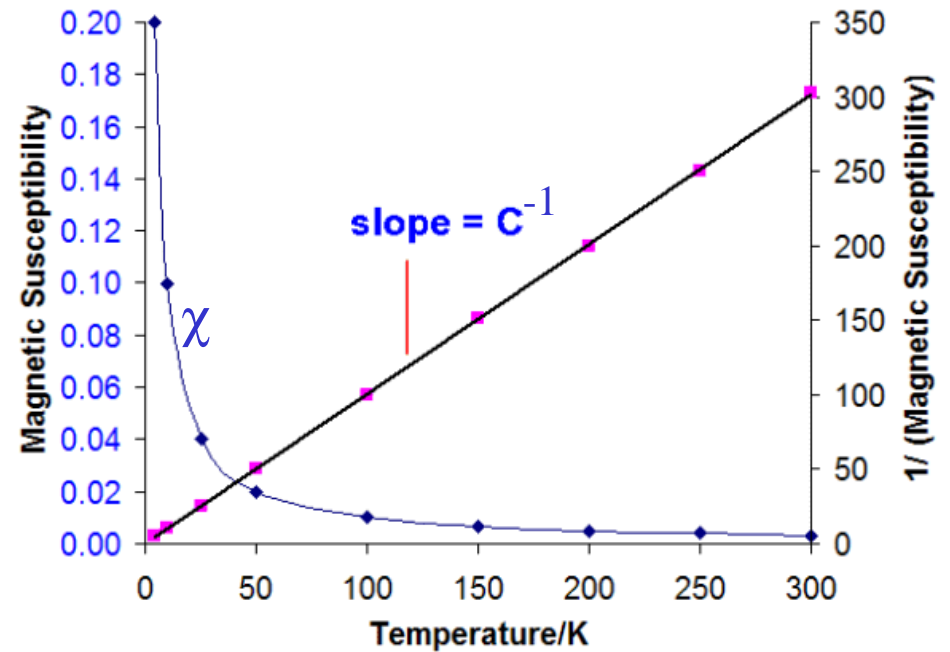
$$\chi^{-1} = \frac{T}{C}$$

Ley de Curie

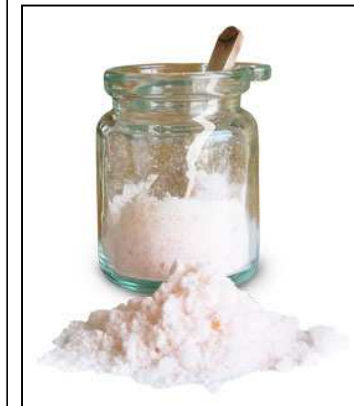
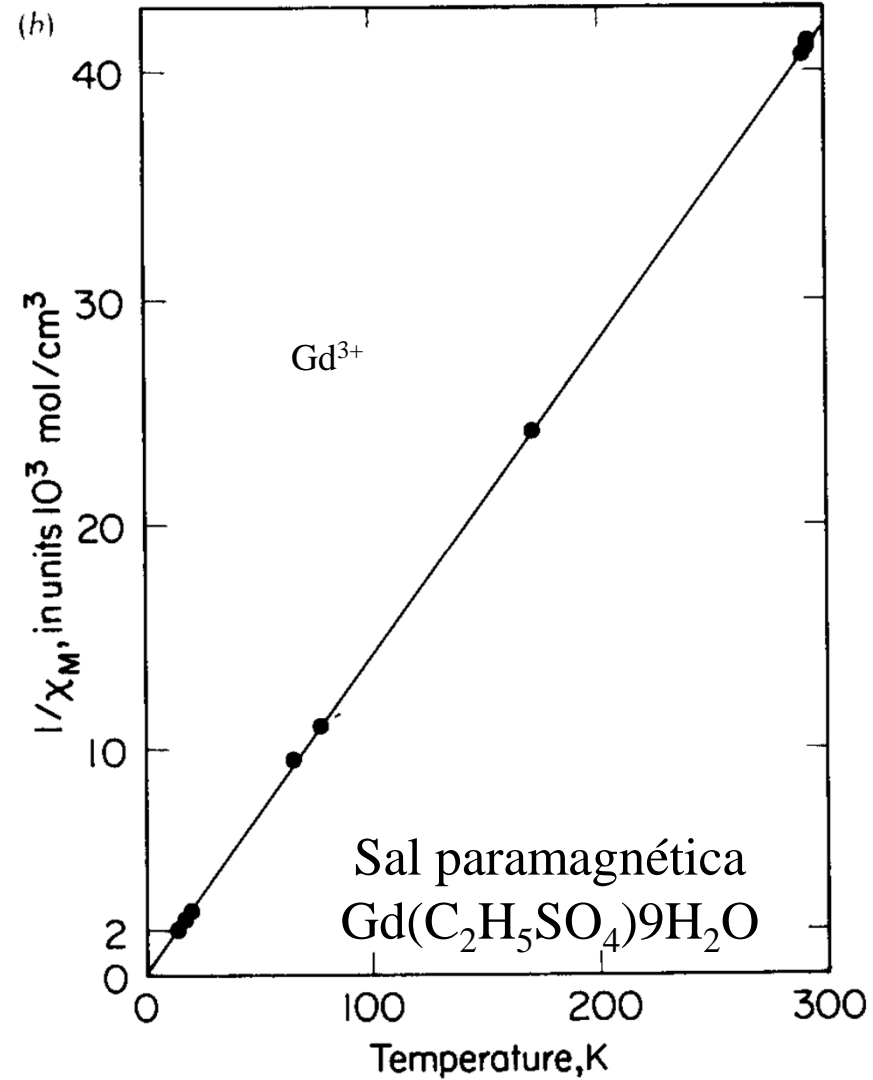
Pierre Curie  
(1859-1906)



Curie Law Plots



## Susceptibilidad inicial



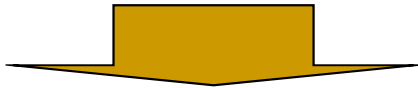
Límite clásico ( $J \rightarrow \infty$ )

$$B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)$$

Cuando  $J \rightarrow \infty$

$$(2J+1)/2J \rightarrow 1$$

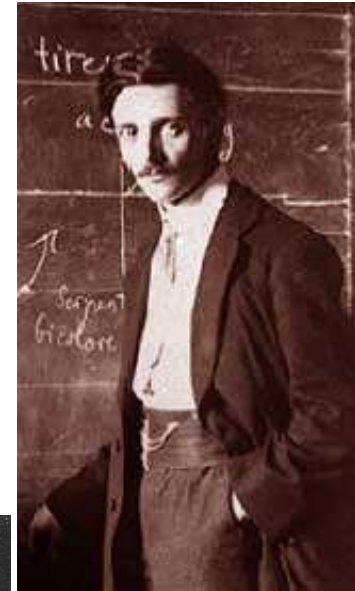
$$\coth(x/2J) \rightarrow 2J/x$$



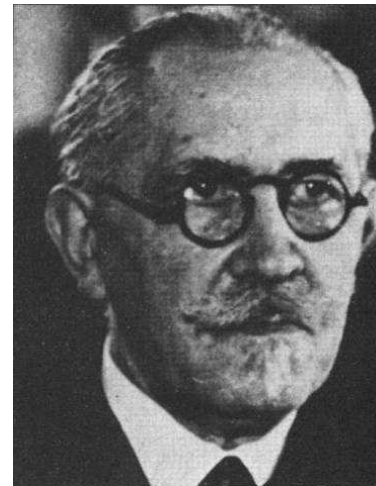
$$B_J(x) \rightarrow L(x) = \coth(x) - \frac{1}{x}$$

Función de  
Langevin

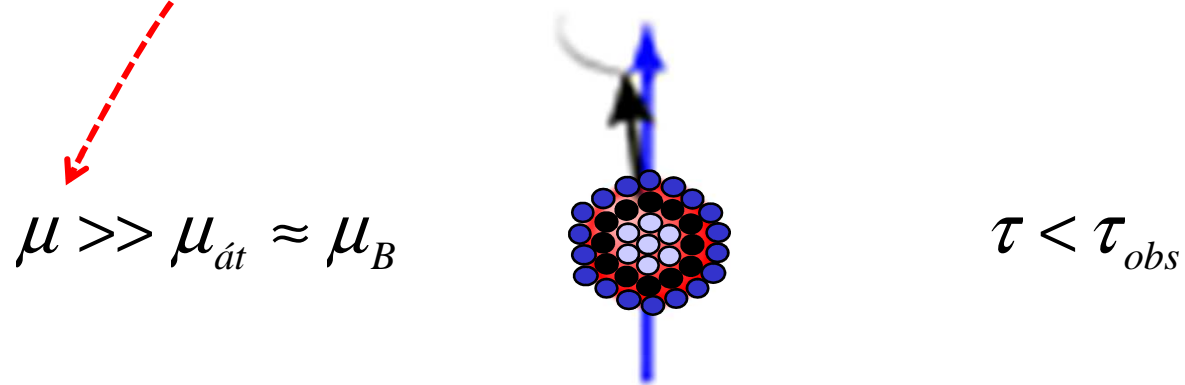
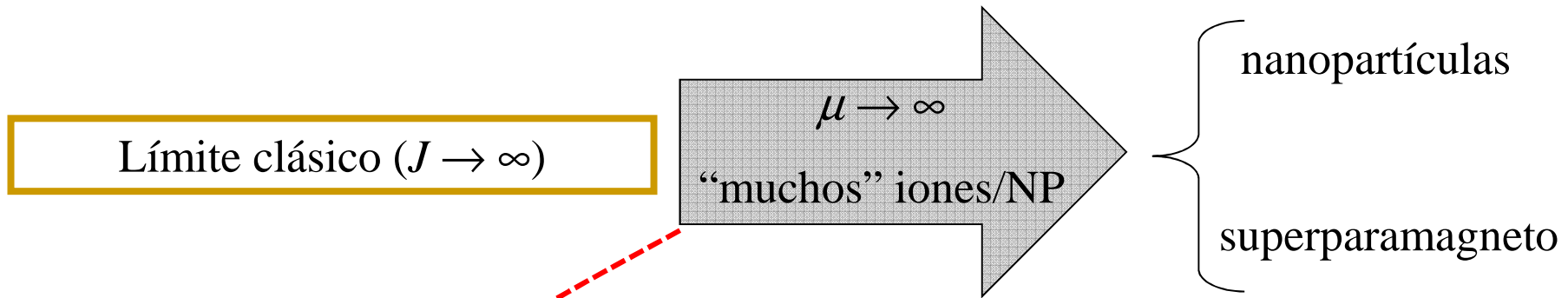
Función de  
Brillouin



**Léon Brillouin**  
(1889-1969)

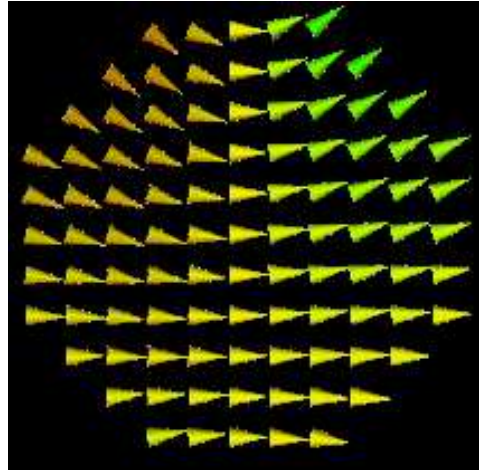


**Paul Langevin**  
(1872-1946)

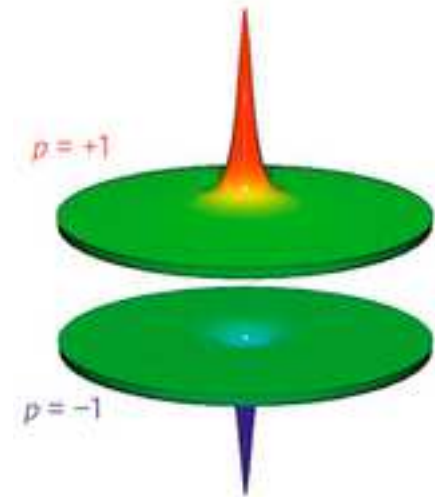
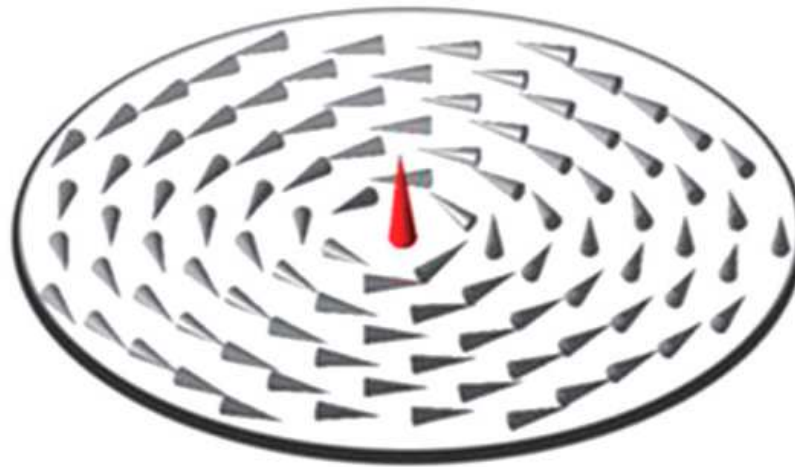
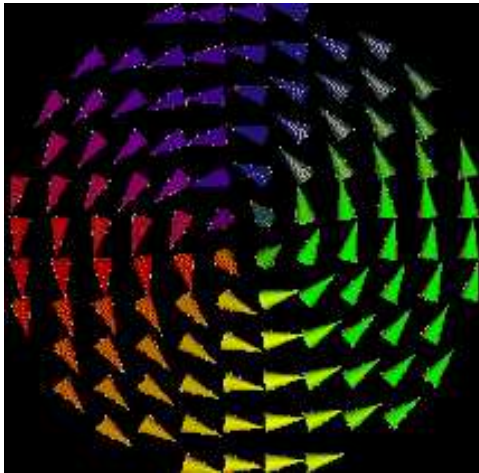


# De NPs monodominio a NPs multidominio

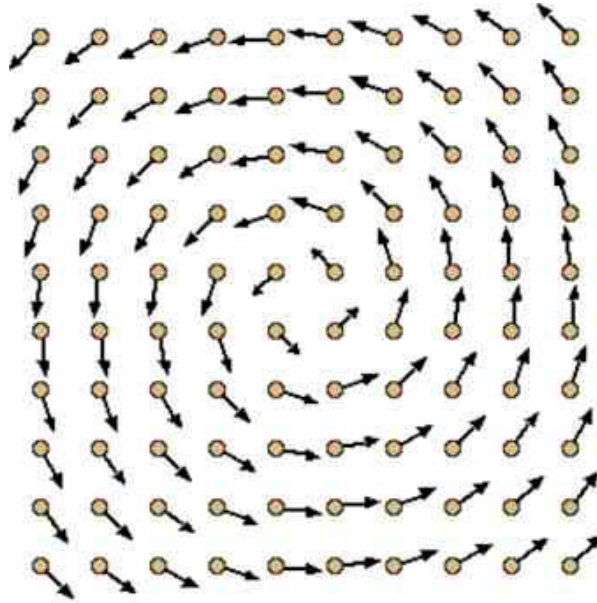
monodominio



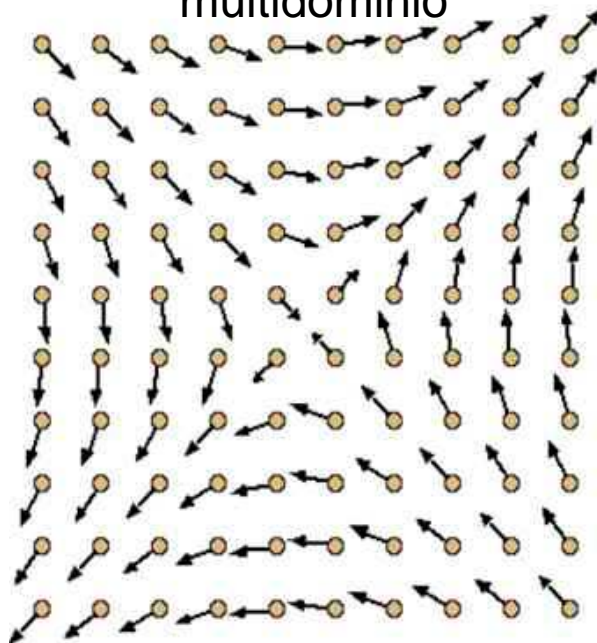
vórtice



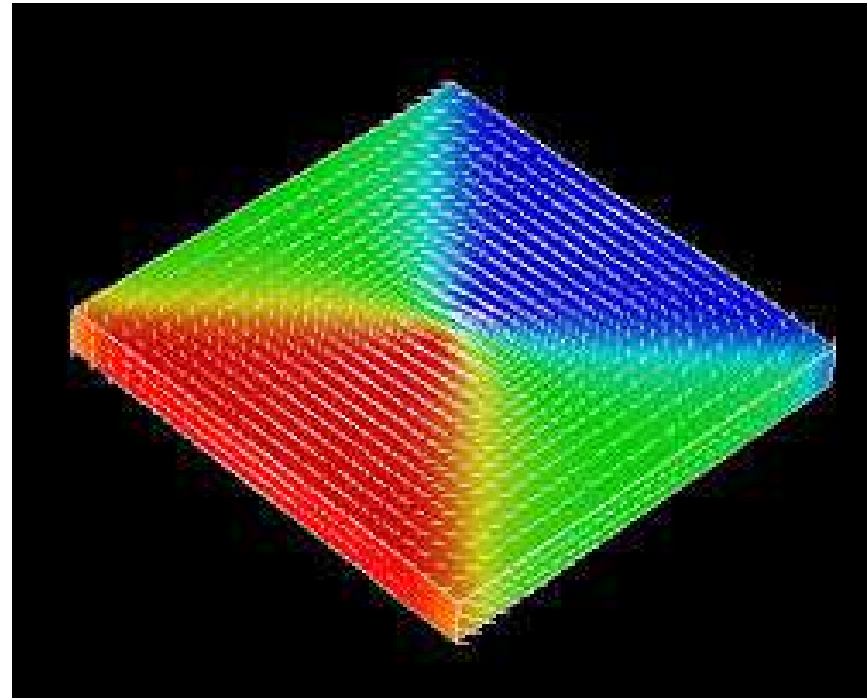
vórtice



multidominio



multidominio



# Estimación del tamaño límite para NPs monodominio

JOURNAL OF APPLIED PHYSICS **108**, 123920 (2010)

## A collective dynamics description of dipolar interactions and the coercive field of magnetic nanoparticles

R. K. Das,<sup>1</sup> S. Rawal,<sup>2</sup> D. Norton,<sup>2</sup> and A. F. Hebard<sup>1,a)</sup>

<sup>1</sup>*Department of Physics, University of Florida, Gainesville, Florida 32611, USA*

<sup>2</sup>*Department of Material Science and Engineering, University of Florida, Gainesville, Florida 32611, USA*

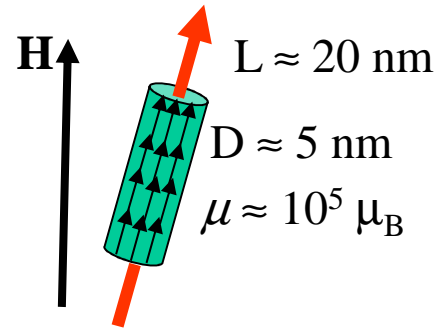
$$\text{Tamaño monodominio máximo: } d_c = 72\sqrt{AK/\mu_0 M_s^2}$$

<sup>9</sup>C. Kittel, *Rev. Mod. Phys.* **21**, 541 (1949).

<sup>10</sup>R. Skomski, *J. Phys.: Condens. Matter* **15**, R841 (2003).



# NPs monodominio, superparamagnetismo

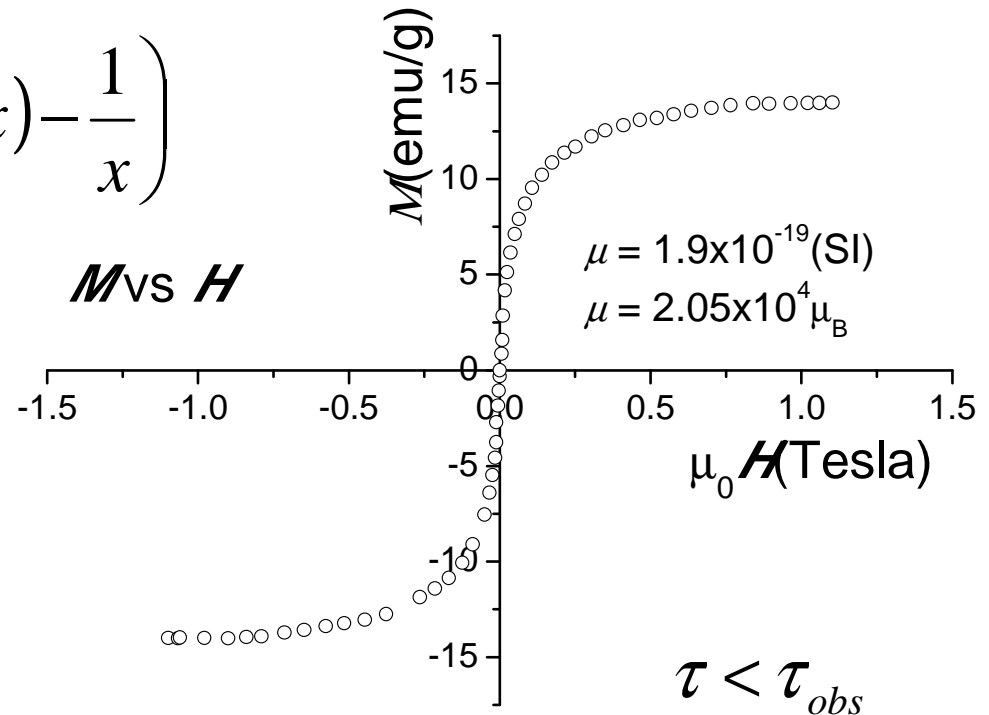


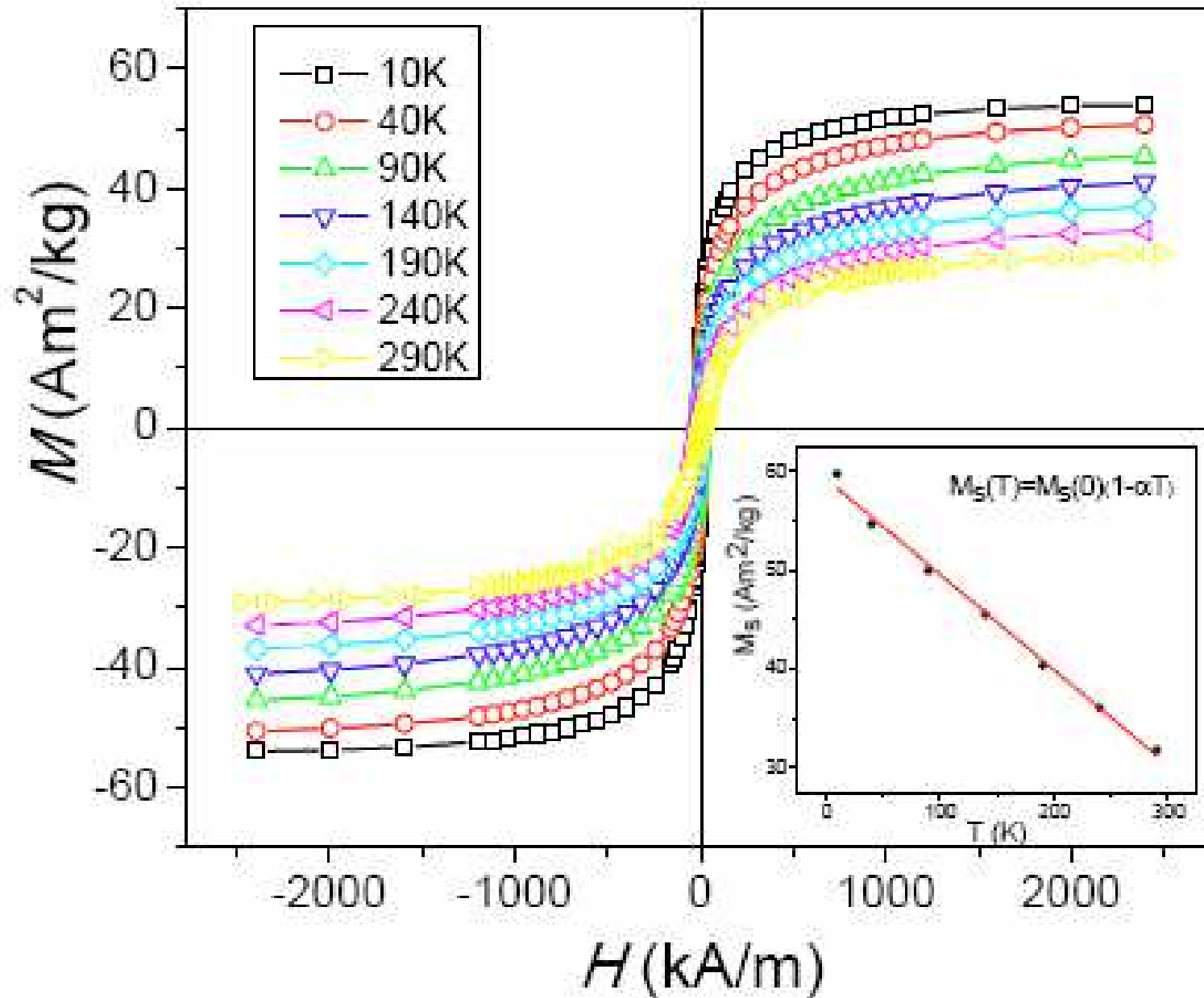
Aerogel de sílica con  
partículas de maghemita  
(~5x20 nm)

$$\langle \mu \rangle_H = \mu L(x) = \mu \left( \coth(x) - \frac{1}{x} \right)$$

$$x = \frac{\mu_0 \mu H}{kT}$$

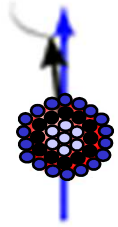
RT





$$\tau < \tau_{obs}$$

Comportamiento superparamagnético de NP de magnetita (4 nm) dispersas en un hidrogel de PVA (ferrogel).



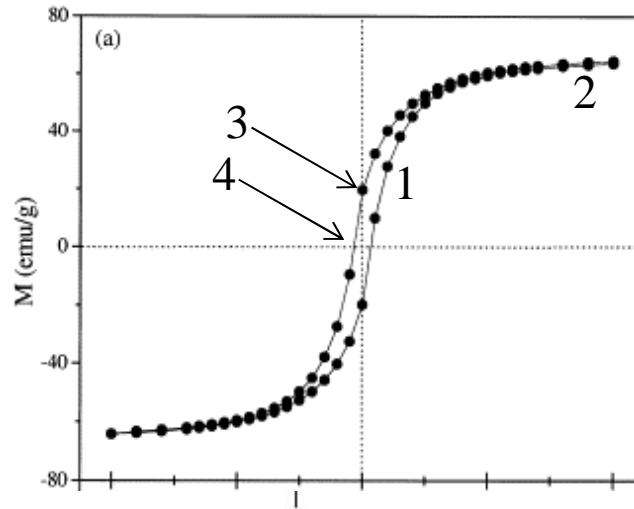
NPs monodominio,  
superparamagnetismo?

$$\tau < \tau_{obs}$$

Cuando consideramos estabilidad de coloides y  
aplicaciones biomédicas, qué es  $\tau_{obs}$ ?

# Regimen bloqueado

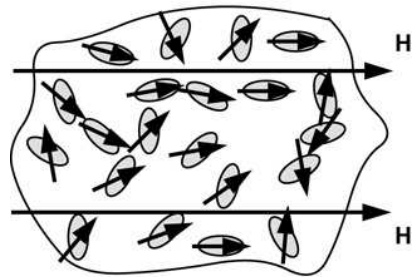
Nanopartículas monodominio



$$\tau > \tau_{obs}$$

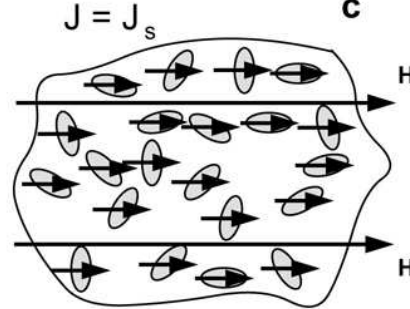
Point 1

**b**



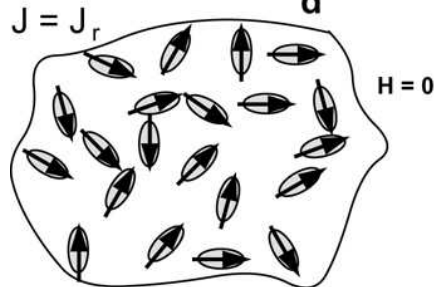
Point 2

**c**



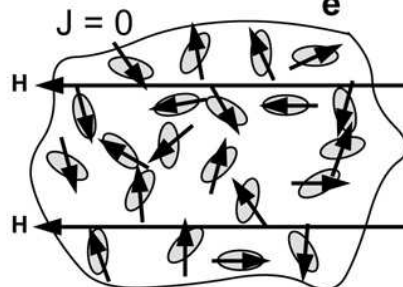
Point 3

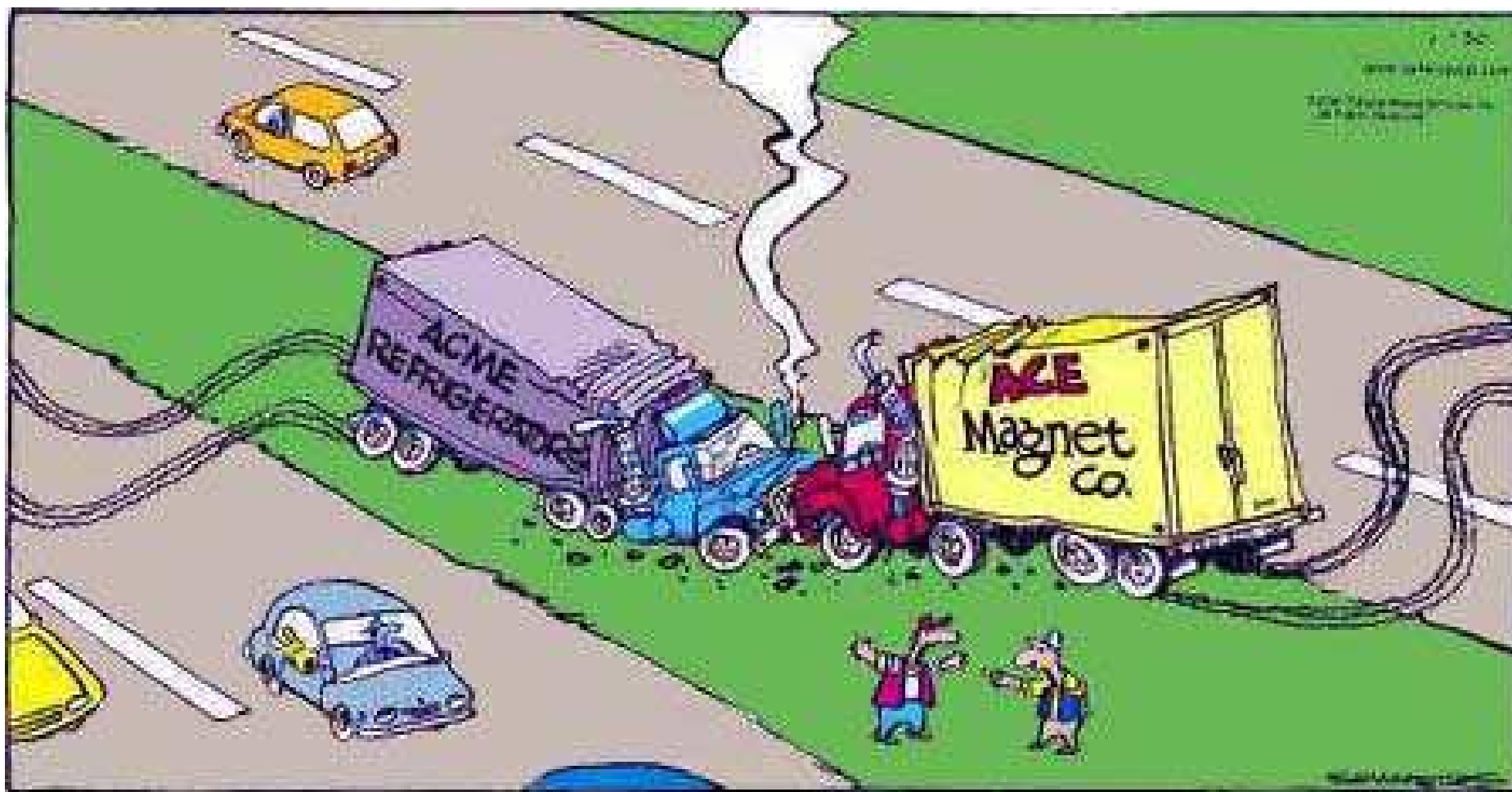
**d**



Point 4

**e**





*Fin módulo*