

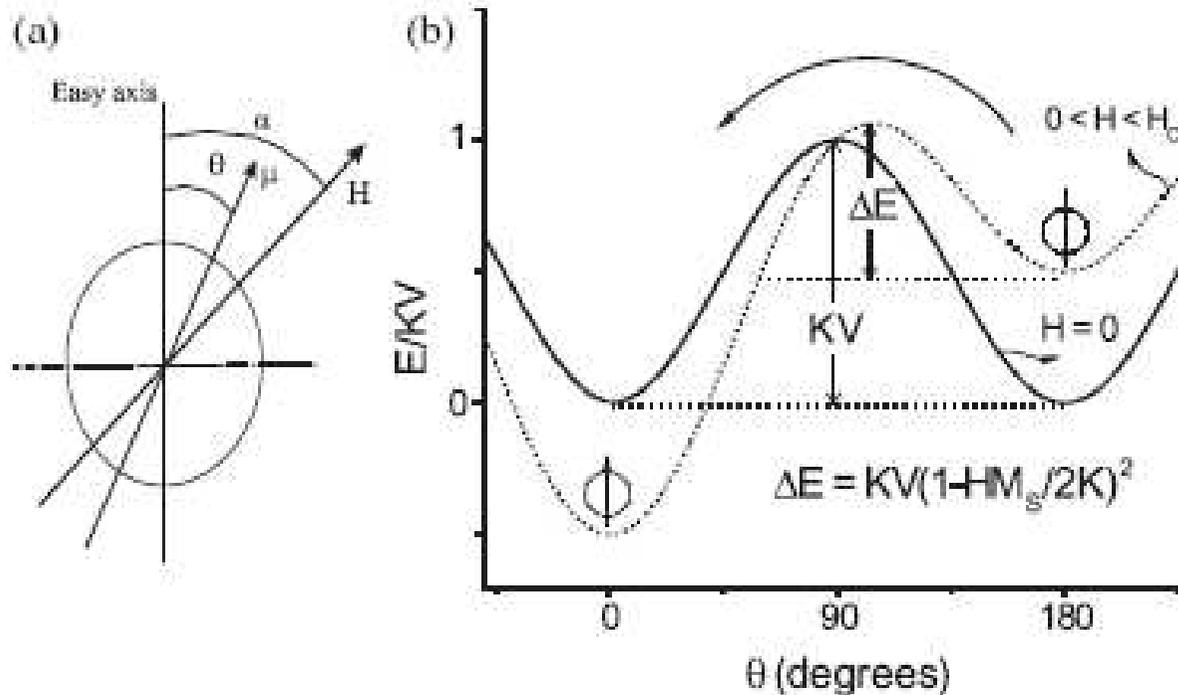
*Algunas consideraciones extra*

# Superparamagnetism and Other Magnetic Features in Granular Materials: A Review on Ideal and Real Systems

M. Knobel<sup>1,\*</sup>, W. C. Nunes<sup>1</sup>, L. M. Socolovsky<sup>2</sup>, E. De Biasi<sup>3</sup>, J. M. Vargas<sup>4</sup>, and J. C. Denardin<sup>5</sup>

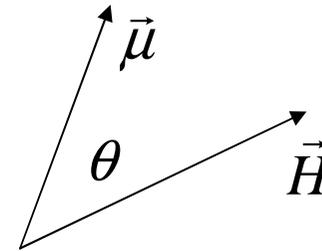
*Journal of  
Nanoscience and Nanotechnology*  
Vol. 8, 2836–2857, 2008

$$E = KV \sin^2 \theta - \mu_0 \mu H \cos(\alpha - \theta)$$



Cuando  $kV \ll \mu_0 \mu H$

$$E = -\mu_0 \mu H \cos \theta$$



Es una expresión idéntica a la del paramagnetismo, sólo que para un momento de NP  $\mu \gg \mu_{\text{át}}$

Por lo que la solución para la magnetización es:

$$\frac{M(H, T)}{M_s} = \coth\left(\frac{\mu_0 \mu H}{kT}\right) - \frac{kT}{\mu_0 \mu H} = L\left(\frac{\mu_0 \mu H}{kT}\right)$$

Función de Langevin

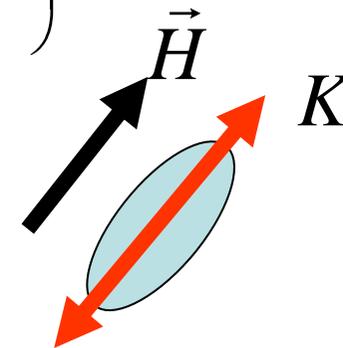
# Simple models for dynamic hysteresis loop calculations of magnetic single-domain nanoparticles: Application to magnetic hyperthermia optimization

J. Carrey,<sup>a)</sup> B. Mehdaoui, and M. Respaud      JOURNAL OF APPLIED PHYSICS **109**, 083921 (2011)

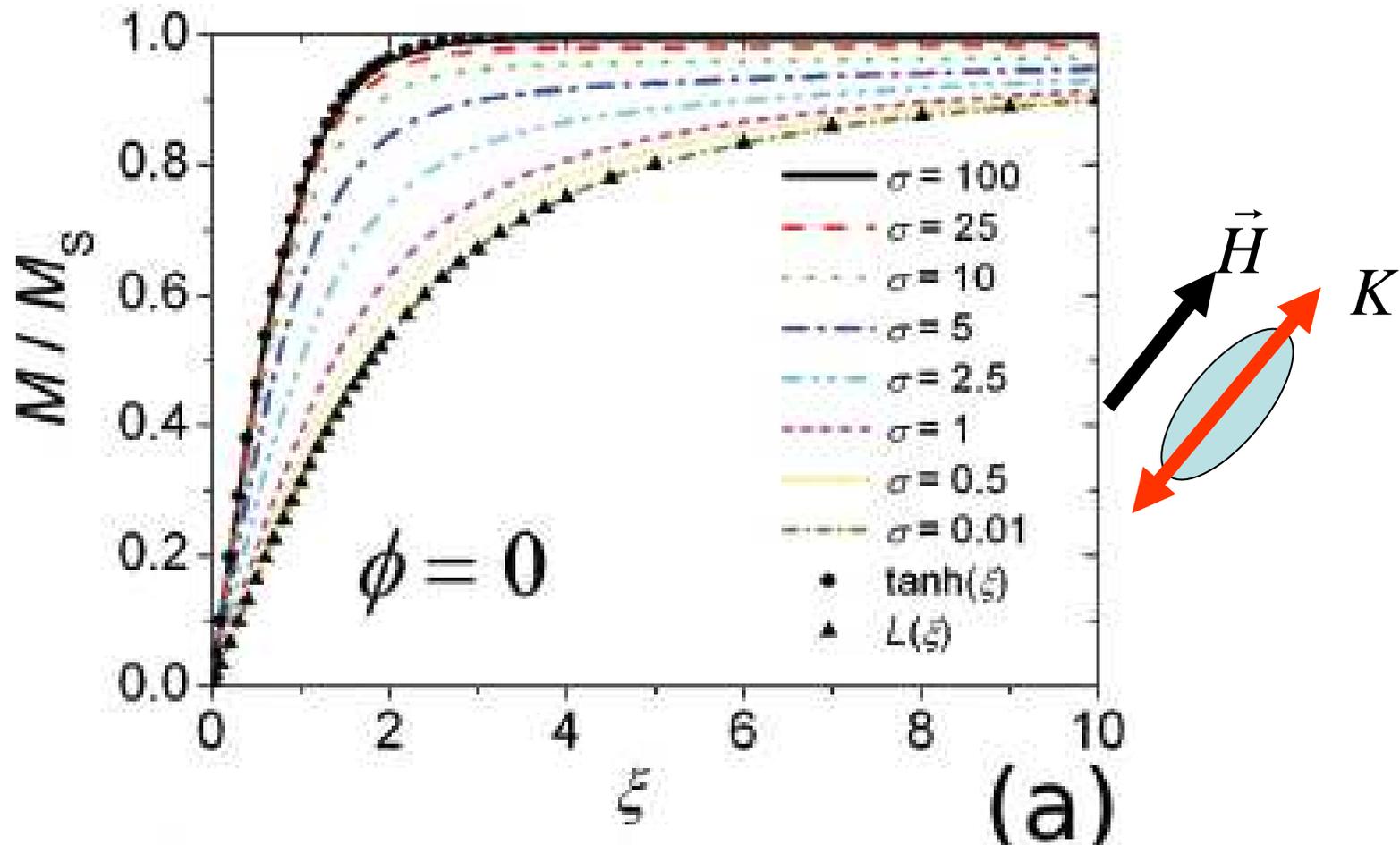
Pero cuando la condición  $KV \ll \mu_0\mu H$  no se cumple, la magnetización presenta una dependencia distinta de H y T. En el caso de que H se aplique en la dirección del eje fácil (K), tenemos:

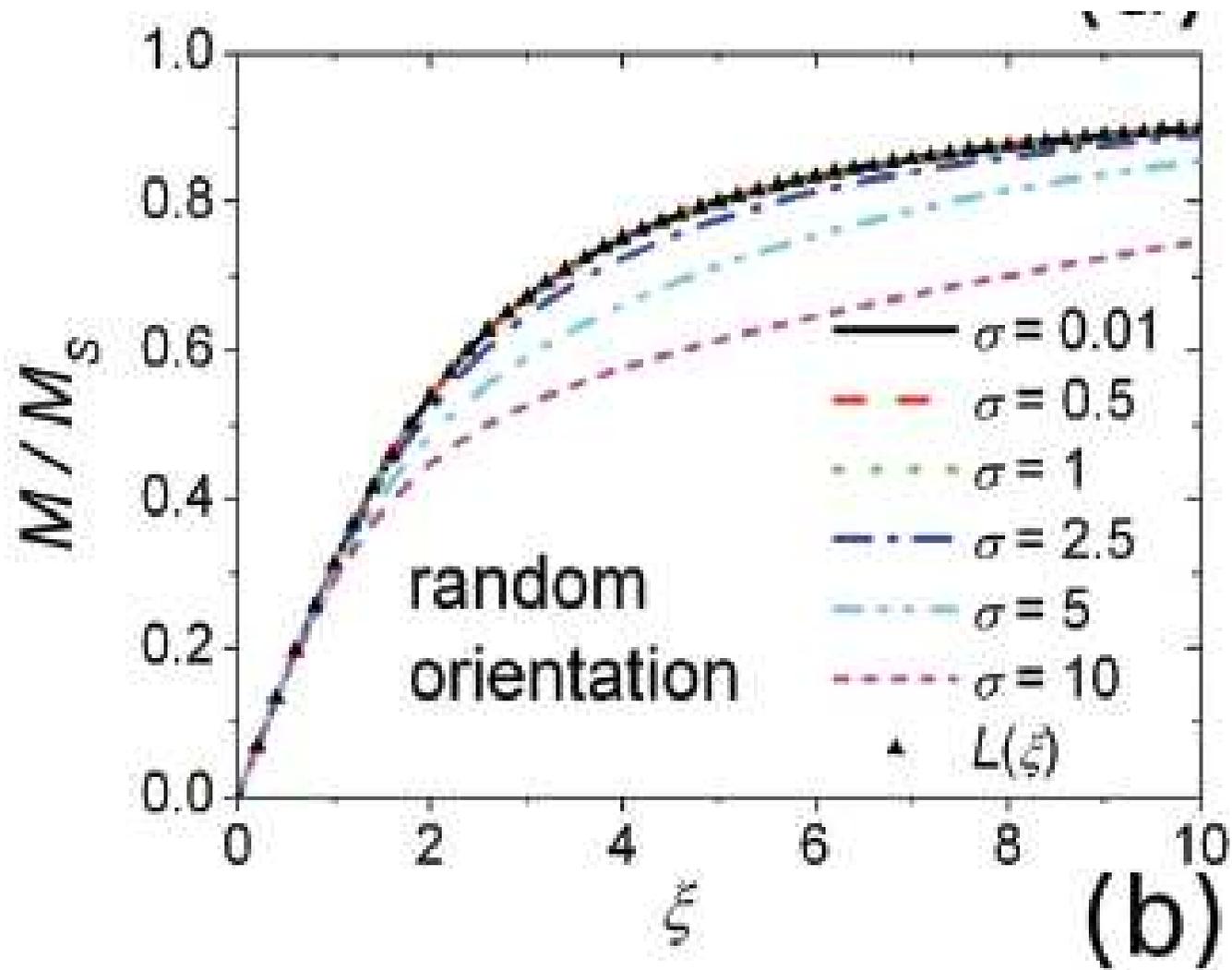
$$\frac{M(H, T)}{M_s} = \frac{\exp\left(\frac{E_K}{kT} + \frac{\mu_0\mu H}{kT}\right) - \exp\left(\frac{E_K}{kT} - \frac{\mu_0\mu H}{kT}\right)}{\exp\left(\frac{E_K}{kT} + \frac{\mu_0\mu H}{kT}\right) + \exp\left(\frac{E_K}{kT} - \frac{\mu_0\mu H}{kT}\right)} = \tanh\left(\frac{\mu_0\mu H}{kT}\right)$$

$$E_K = KV \left(1 - \frac{H}{H_K}\right)^2$$



$$\frac{M(H,T)}{M_s} = m\left(\frac{\mu_0\mu H}{kT}\right) = \begin{cases} Lan\left(\frac{\mu_0\mu H}{kT}\right) & KV \ll \mu_0\mu H \quad \text{random} \\ Tanh\left(\frac{\mu_0\mu H}{kT}\right) & KV \approx \geq \mu_0\mu H \quad \text{aligned} \end{cases}$$





distribución aleatoria de orientaciones de ejes fáciles

## Susceptibilidad dc

distribución aleatoria de  
orientaciones de ejes fáciles

$$\chi_{sp} = \chi_{Lan} \approx \frac{\mu_0 M_S^2 V}{3k_B T} \quad \begin{array}{l} KV \leq \mu_0 \mu H \\ KV \geq \mu_0 \mu H \end{array}$$

Campo en la dirección del eje  
fácil

$$\chi_{sp} = \chi_{\sigma} \approx \frac{\mu_0 M_S^2 V}{k_B T} \quad \begin{array}{l} KV \approx \mu_0 \mu H \\ KV \geq \mu_0 \mu H \end{array} \quad \sigma = \frac{KV}{kT}$$

$$\frac{\chi_{\sigma}}{\chi_{Lan}} \approx 3 - \frac{2}{1 + \left(\frac{\sigma}{3.4}\right)^{1.47}}$$

## Susceptibilidad ac

$$\chi_{sp} = \chi' - i\chi''$$

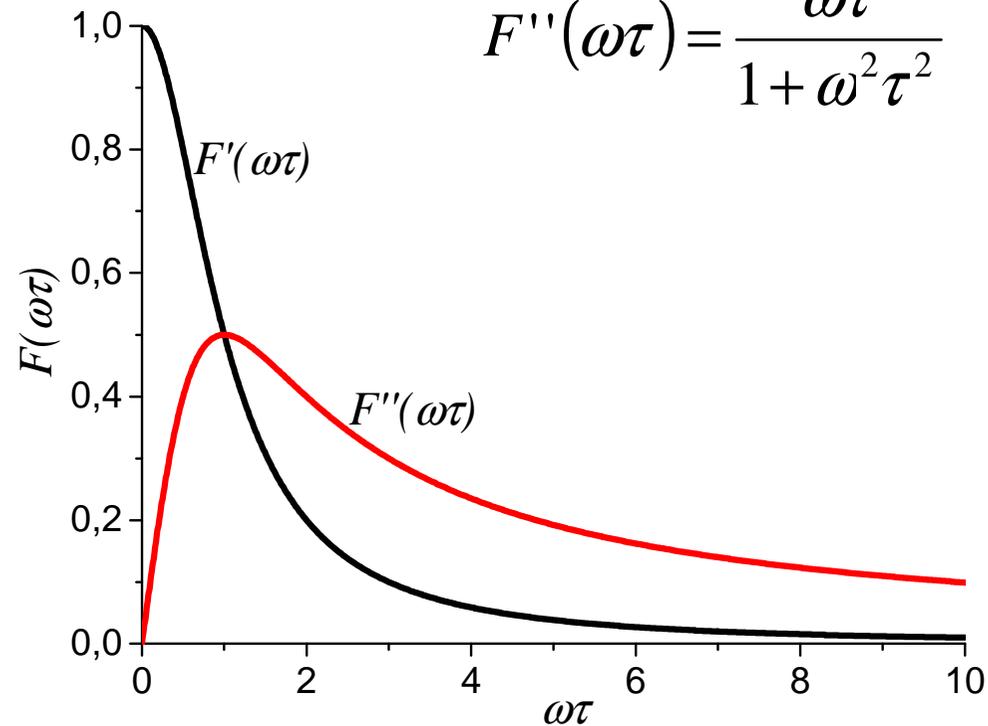
$$\chi_{sp}^{est} = \chi_{Lan} \approx \frac{\mu_0 M_S^2 V}{3k_B T}$$

$$\left. \begin{matrix} \chi' \\ \chi'' \end{matrix} \right\} \approx \chi_{sp}^{est} \begin{cases} F'(\omega\tau) \\ F''(\omega\tau) \end{cases}$$

$$F'(\omega\tau) = \frac{1}{1 + \omega^2 \tau^2}$$

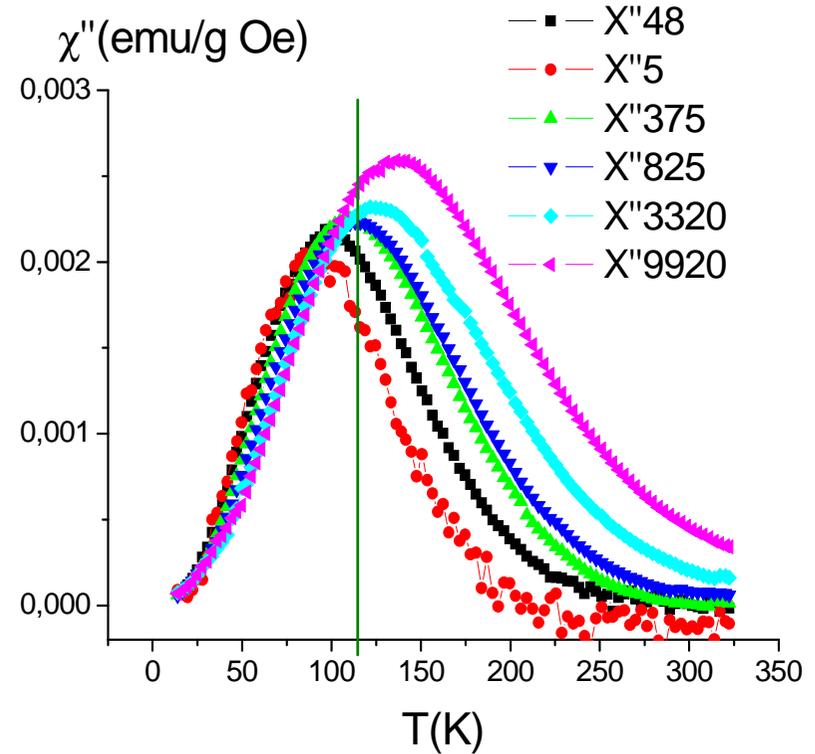
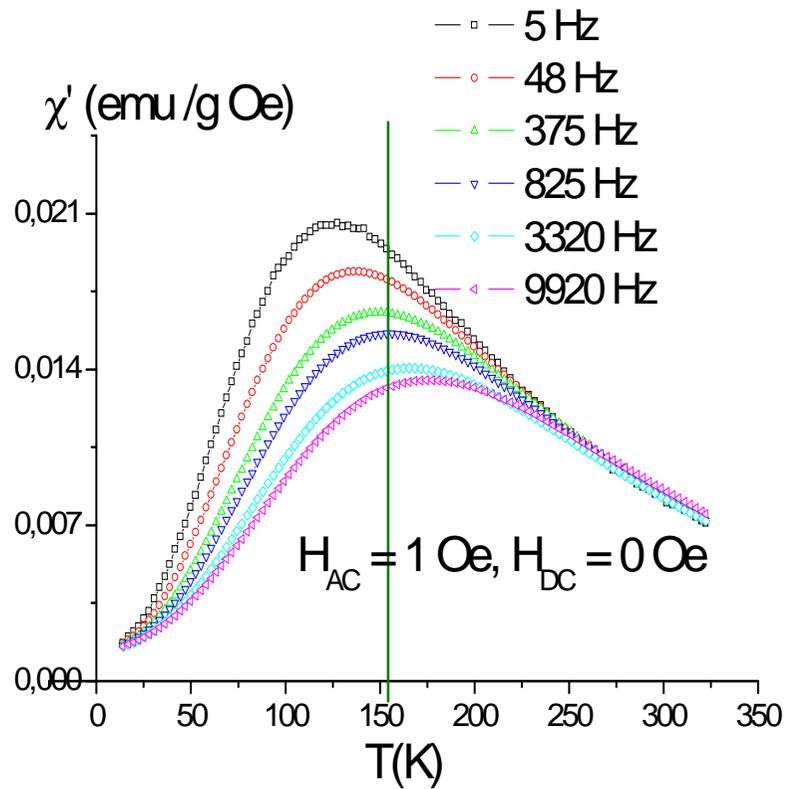
$$\tau = \tau_0 e^{KV/kT}$$

$$F''(\omega\tau) = \frac{\omega\tau}{1 + \omega^2 \tau^2}$$



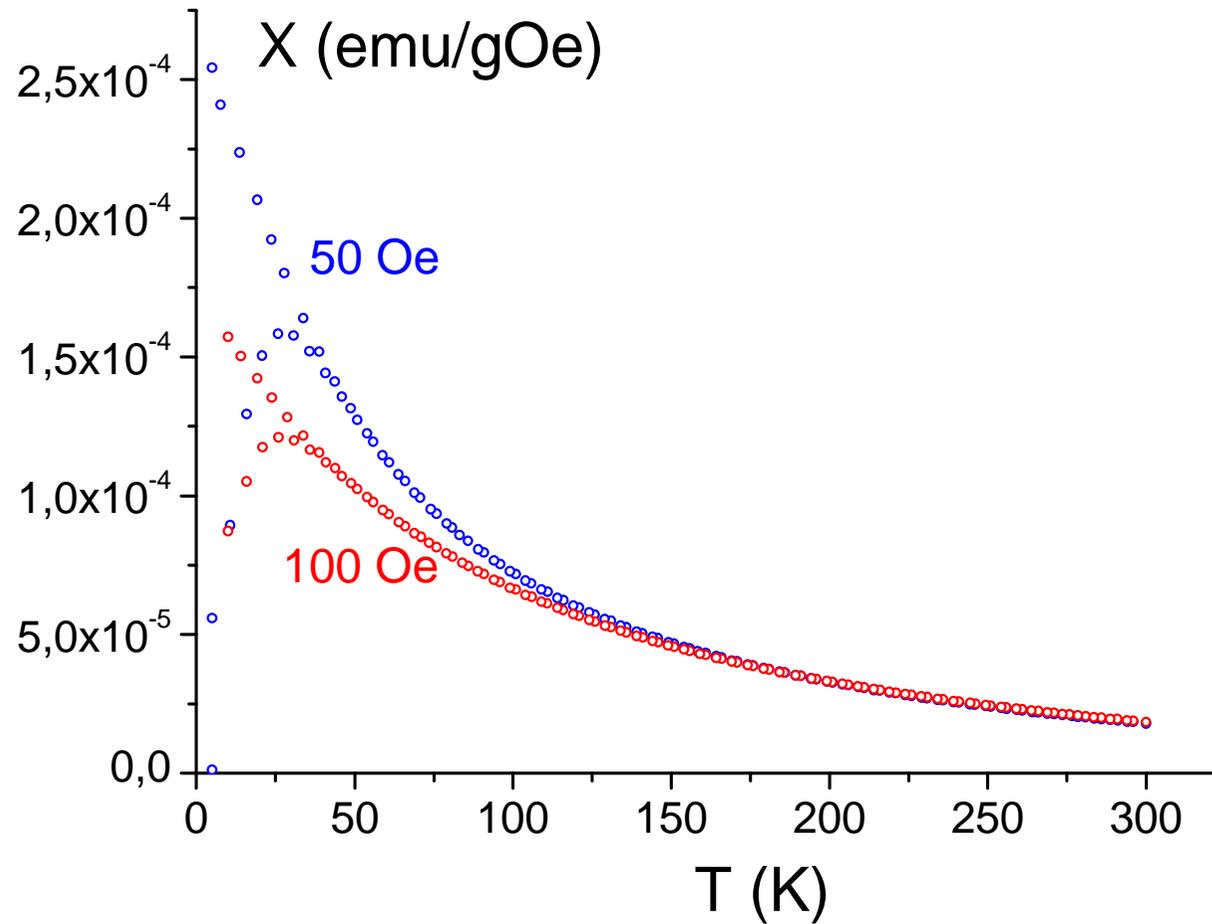
# Susceptibilidad ac

$$\left. \begin{matrix} \chi' \\ \chi'' \end{matrix} \right\} \approx \frac{\mu_0 M_S^2 V}{3k_B T} \begin{cases} \frac{1}{1 + \omega^2 \tau^2} \\ \frac{\omega \tau}{1 + \omega^2 \tau^2} \end{cases} \quad \tau = \tau_0 e^{KV/kT}$$



dc ZFC – FC

$$\omega \approx \frac{1}{\tau_{\text{exp}}} \quad \longrightarrow \quad \chi \approx \frac{\mu_0 M_S^2 V}{3k_B T} \frac{1}{1 + (\tau / \tau_{\text{exp}})^2}$$



$$H_c(T) \approx 0.48H_K \left( 1 - \sqrt{\frac{kT}{KV} \ln(\tau_{\text{exp}} / \tau_0)} \right)$$

distribución aleatoria de orientaciones de ejes fáciles

$$H_c(T) \approx H_K (1 - \sqrt{\kappa})$$

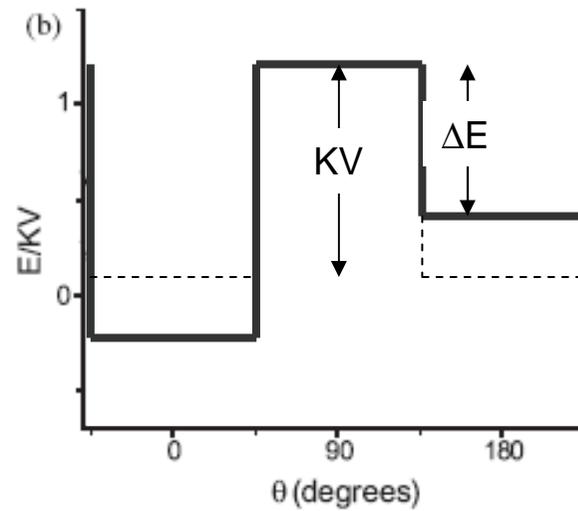
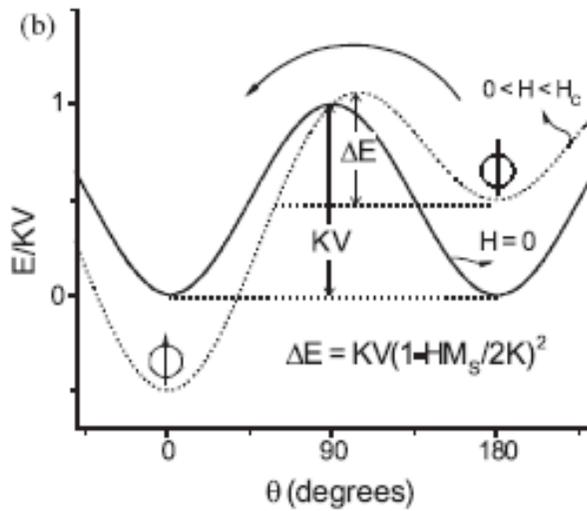
Campo en la dirección del eje fácil

$$\kappa \approx \frac{kT}{KV} \ln \left( \frac{kT}{4\mu_0 H_{\text{max}} M_S V f \tau_0} \right)$$

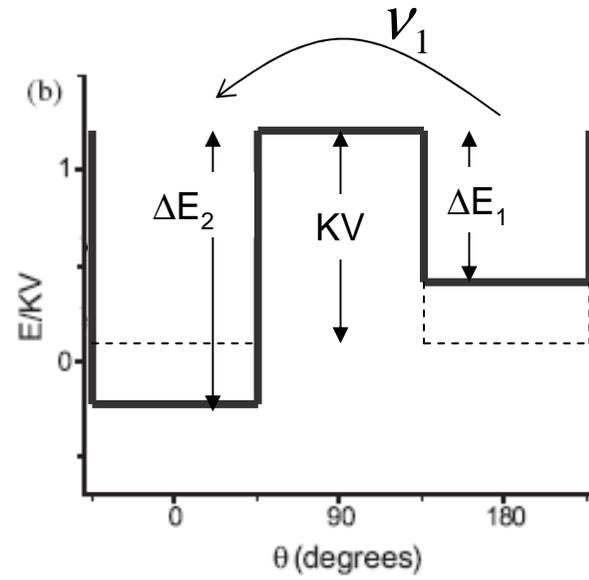
N. A. Usov and Y. B. Grebenshchikov, J. Appl. Phys. 106, 023917, (2009).

# Modelo de dos niveles

Campo en la dirección del eje fácil



simplificación



$$\nu_1 \approx \nu_0 e^{\Delta E_1/kT} \quad \nu_2 \approx \nu_0 e^{\Delta E_2/kT}$$

$$\frac{\partial p_1}{\partial t} \approx (1 - p_1)\nu_2 - p_1\nu_1$$

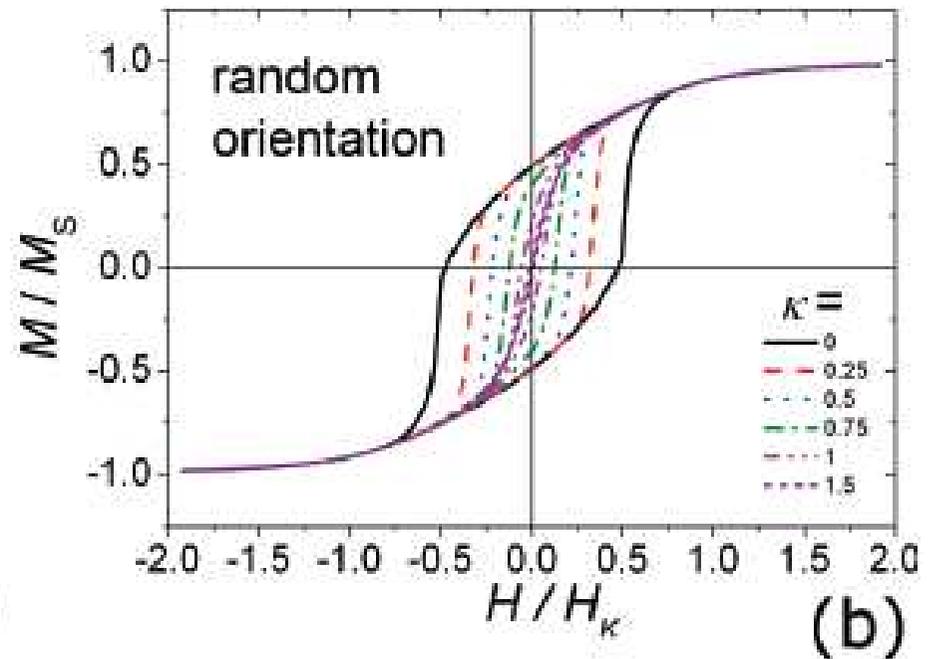
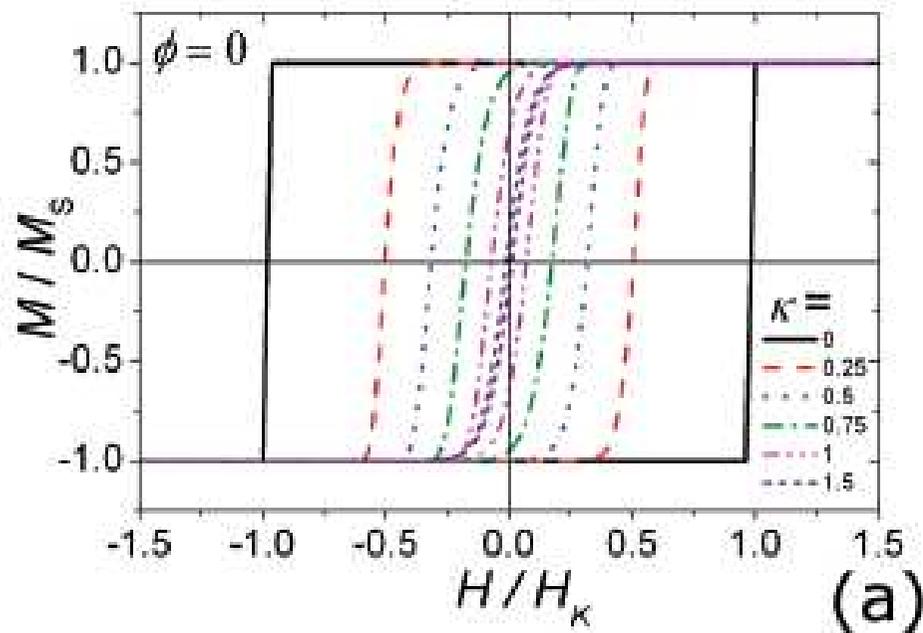
$$M = M_s(2p_1 - 1)$$

$$p_1 = \frac{1}{2} \left( \frac{M}{M_s} + 1 \right)$$

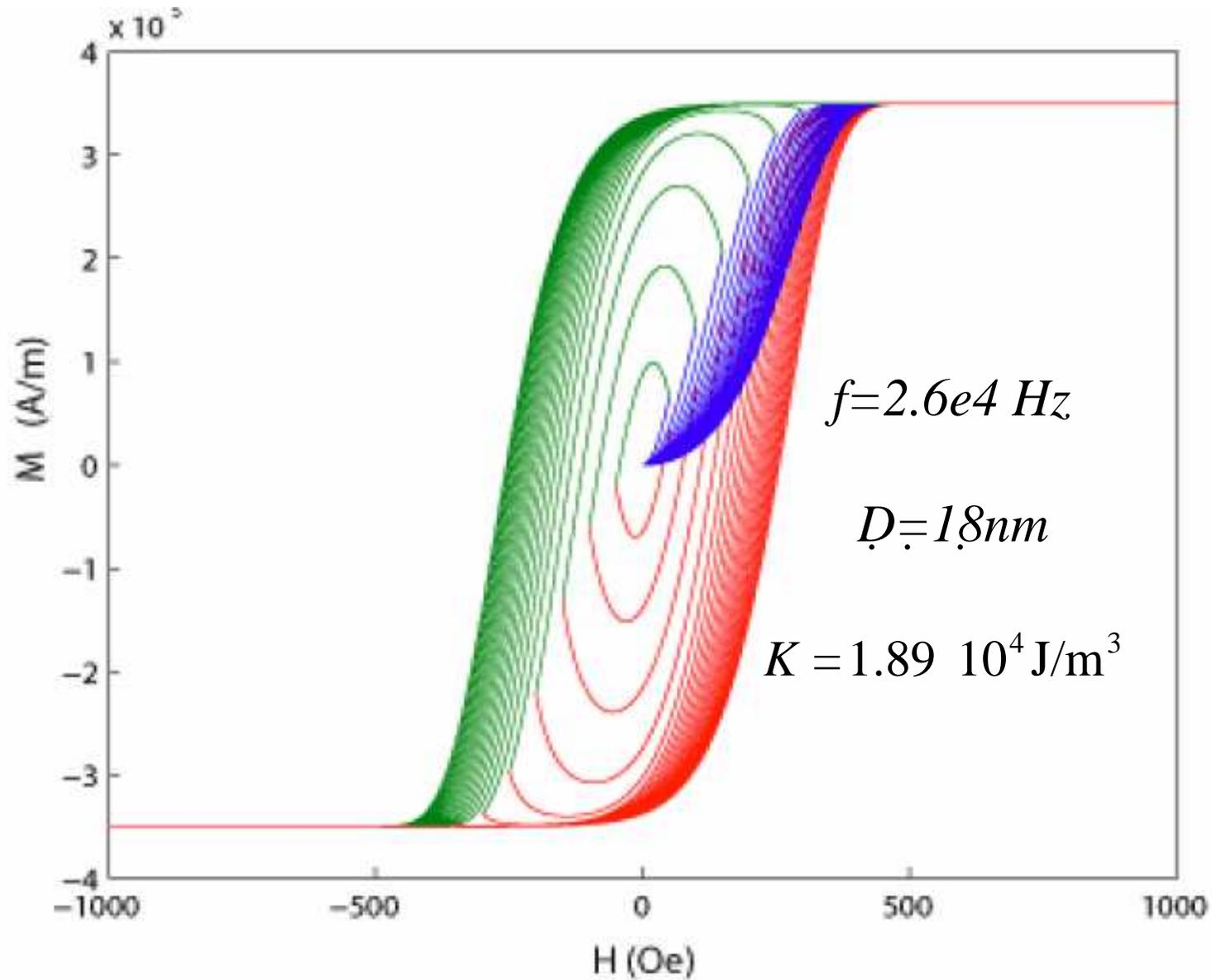
$$\frac{\partial M}{\partial t} = 2M_s \frac{\partial p_1}{\partial t} = 2M_s \left[ \left( 1 - \frac{1}{2} \left( \frac{M}{M_s} + 1 \right) \right) v_2 - \frac{1}{2} \left( \frac{M}{M_s} + 1 \right) v_1 \right]$$

$$\frac{\partial M}{\partial t} = M_s (v_2 - v_1) \left( 1 - \frac{M}{M_s} \right)$$

$$\text{si } \frac{\partial H}{\partial t} = H_0 f_H(t) \Rightarrow dM = \frac{M_s}{H_0} \left( 1 - \frac{M}{M_s} \right) \frac{(v_2 - v_1)}{f_H(t)} dH$$



## Comportamiento a alta frecuencia



## Distribución de tamaños

$$f(\mu)d\mu = \frac{1}{\mu\sqrt{2\pi\sigma}} e^{-\frac{\ln^2(\mu/\mu_0)}{2\sigma^2}} d\mu$$

lognormal

$$M(H, T) = N \frac{\int_0^\infty \mu f(\mu) m\left(\frac{\mu_0 \mu H}{kT}\right) d\mu}{\int_0^\infty f(\mu) d\mu}$$

$N$  Nro NP/volumen

Multiplicando y dividiendo por  $\int_0^\infty \mu f(\mu) d\mu$

$$M(H, T) = N \langle \mu \rangle_f \frac{\int_0^\infty \mu f(\mu) m\left(\frac{\mu_0 \mu H}{kT}\right) d\mu}{\int_0^\infty \mu f(\mu) d\mu}, \quad \langle \mu \rangle_f = \frac{\int_0^\infty \mu f(\mu) d\mu}{\int_0^\infty f(\mu) d\mu}$$

Que puede escribirse

$$M(H, T) = N \langle \mu \rangle_f \frac{\int_0^\infty g(\mu) m \left( \frac{\mu_0 \mu H}{kT} \right) d\mu}{\int_0^\infty g(\mu) d\mu}$$

$f(\mu)d\mu$ , *dist. número*

$g(\mu)d\mu$ , *dist. volumen*