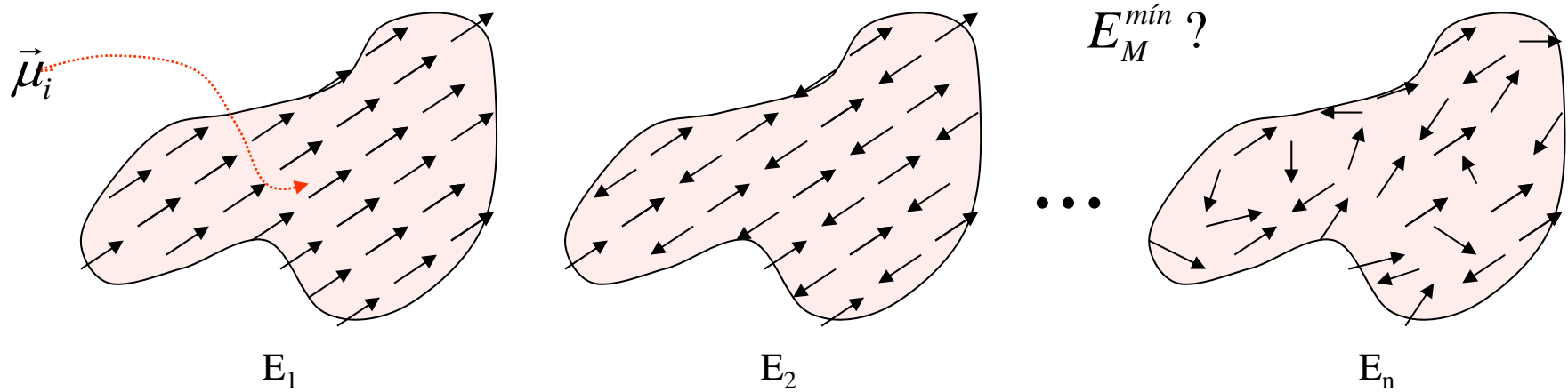


Energía magnetostática – existencia de dominios

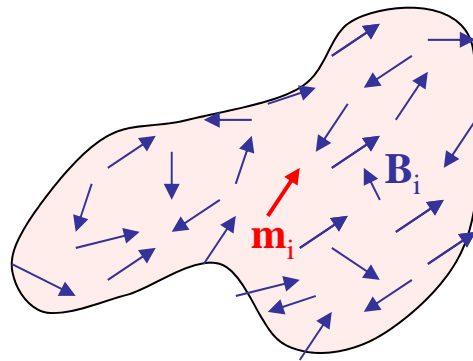
# Energía magnetostática

Energía de interacción entre los dipolos de un material magnetizado



$$E_M = -\frac{1}{2} \sum_i \vec{\mu}_i \cdot \vec{B}_i = -\frac{\mu_0}{2} \sum_i \vec{\mu}_i \cdot \vec{H}_i = -\frac{\mu_0}{2} \sum_i \vec{M}_i \cdot \vec{H}_i V_i \approx -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV$$

$$\vec{B}_i = \sum_{j \neq i} \vec{B}_j(\vec{r}_i)$$



Evaluación de:

$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV$$

Dado un cuerpo (forma, volumen  $V$ , superficie  $S$ ),



Una distribución de magnetización  $\vec{M}$ ,

y las ecuaciones de Maxwell:

$$\begin{aligned} \nabla \cdot \vec{B} &= 0, \\ \nabla \times \vec{E} + \partial \vec{B} / \partial t &= 0, \\ \nabla \cdot \vec{D} &= \rho, \\ \nabla \times \vec{H} - \partial \vec{D} / \partial t &= \vec{j}. \end{aligned}$$

En ausencia de corrientes:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = 0$$

además

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{H} = -\nabla U$$

Potencial escalar

$U$  continuo

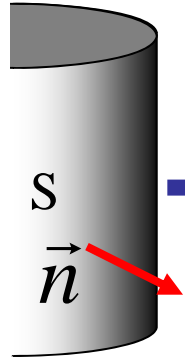
$$\nabla \cdot (-\nabla U + \vec{M}) = 0$$

$$\nabla^2 U_{\text{int}} = \nabla \cdot \vec{M}$$

$$\nabla^2 U_{\text{out}} = 0$$

$M(\vec{r})$  dada

condiciones de contorno



$$U_{\text{int}}|_S = U_{\text{ext}}|_S$$
$$\frac{\partial U_{\text{int}}}{\partial n}\bigg|_S - \frac{\partial U_{\text{ext}}}{\partial n}\bigg|_S = \vec{M} \cdot \vec{n}$$

$$\begin{aligned} |rU| & \text{ Acotadas para } r \\ |r^2U| & \rightarrow \infty \end{aligned}$$

+

$$\begin{aligned} \nabla^2 U_{\text{int}} &= \vec{\nabla} \cdot \vec{M} \\ \nabla^2 U_{\text{ext}} &= 0 \end{aligned}$$

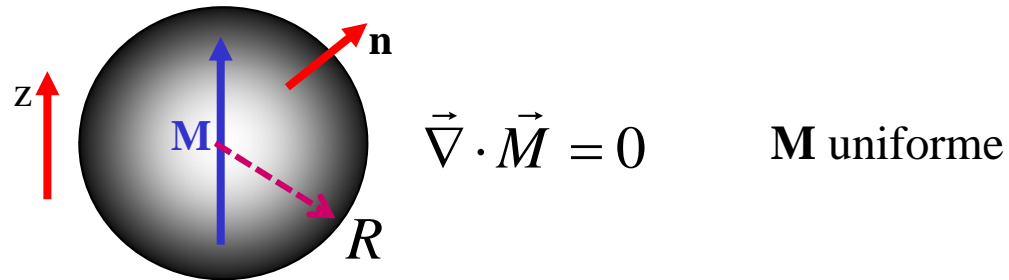
Se puede demostrar que el problema tiene solución única

$$U(\vec{r})$$

$$\vec{H} = -\vec{\nabla} U$$

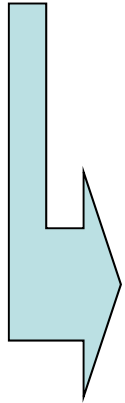
$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV$$

Ejemplo: Esfera de radio R magnetizada uniformemente



$$U(\vec{r}) = \frac{M_S}{3} \times \begin{cases} z, & \text{si } r \leq R \\ \frac{zR^3}{r^3} & \text{si } r > R \end{cases} \Rightarrow \begin{cases} U_{\text{int}} = \frac{M_S}{3} z \\ \vec{H}_{\text{int}} = -\vec{\nabla} U_{\text{int}} \end{cases} \Rightarrow \begin{cases} H_{\text{int}_x} = H_{\text{int}_y} = 0 \\ H_{\text{int}_z} = -\frac{M_S}{3} \end{cases}$$

$$U_{ext} = \frac{M_S R^3 z}{3r^3} = \frac{M_S R^3 z}{3(x^2 + y^2 + z^2)^{3/2}}$$



$$H_{ext_x} = \frac{M_S R^3 xz}{r^5}$$

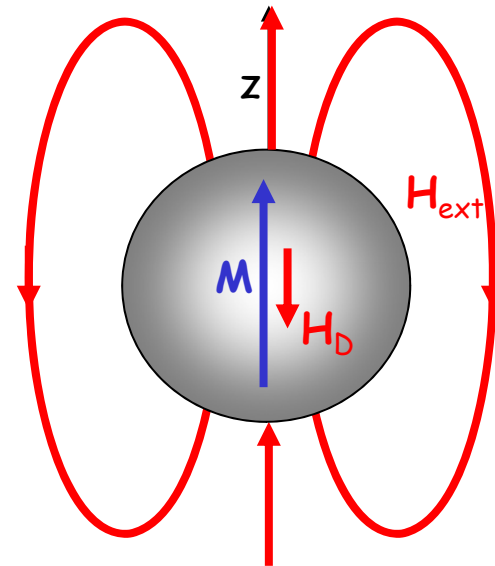
$$H_{ext_y} = \frac{M_S R^3 yz}{r^5}$$

$$H_{ext_z} = \frac{M_S R^3}{r^3} \left( \frac{z^2}{r^2} - \frac{1}{3} \right)$$



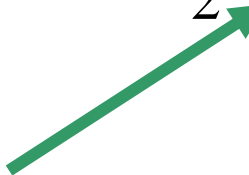
$$H_{int_x} = H_{int_y} = 0$$

$$H_{int_z} = -\frac{M_S}{3}$$



energía

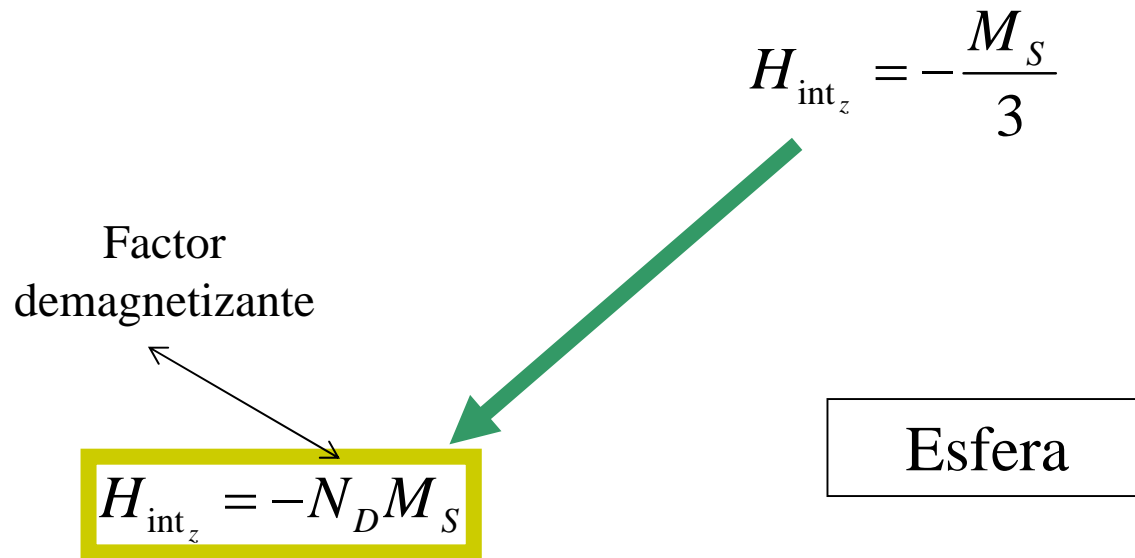
$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV = -\frac{\mu_0}{2} M_S H_{\text{int}_z} V_{\text{esfera}} = \frac{\mu_0}{2} M_S \frac{M_S}{3} \frac{4\pi R^3}{3} = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

$$H_{\text{int}_z} = -\frac{M_S}{3}$$


$$E_M = \frac{2\pi R^3 \mu_0 M_S^2}{9}$$

generalizando  
para otras  
geometrías y  
distribuciones  
de la  
magnetización

$$E_M = C \mu_0 M_S^2$$



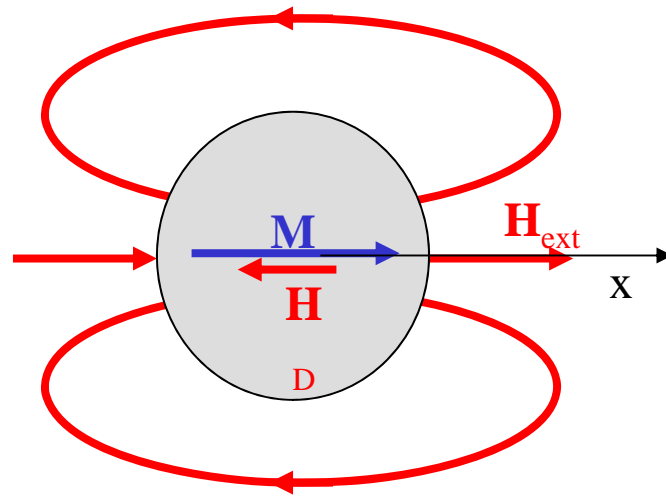
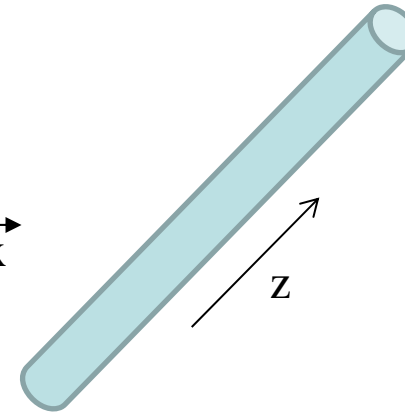
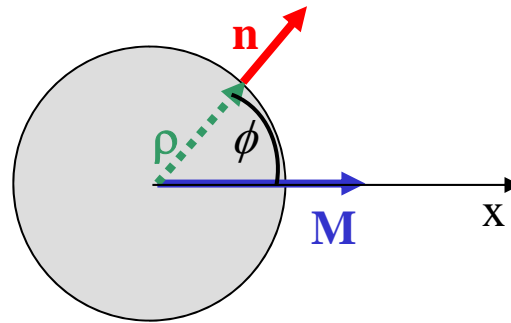
si

$$\begin{cases} \vec{M} = M_S \check{x} \Rightarrow N_x = 1/3 \\ \vec{M} = M_S \check{y} \Rightarrow N_y = 1/3 \\ \vec{M} = M_S \check{z} \Rightarrow N_z = 1/3 \end{cases}$$

$N_x + N_y + N_z = 1$




Cilindro infinito magnetizado uniformemente en dirección perpendicular al eje




$$\begin{cases} H_{\text{int}_z} = H_{\text{int}_y} = 0 \\ H_{\text{int}_x} = -\frac{M_S}{2} \end{cases}$$

Energía magnetostática por unidad de área  $\perp$  al eje del cilindro

$$\varepsilon_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dS = -\frac{\mu_0}{2} M_S \frac{M_S}{2} \pi R^2 = \frac{\pi R^2 \mu_0 M_S^2}{4}$$

$$H_{\text{int}_x} = -\frac{M_S}{2}$$


$$\varepsilon_M = \frac{\pi R^2 \mu_0 M_S^2}{4}$$


de la forma

$$\varepsilon_M = C \mu_0 M_S^2$$


$$H_{\text{int}_x} = -\frac{M_S}{2}$$

de la forma

Factor  
demagnetizante

$$H_{\text{int}_z} = -N_D M_S$$

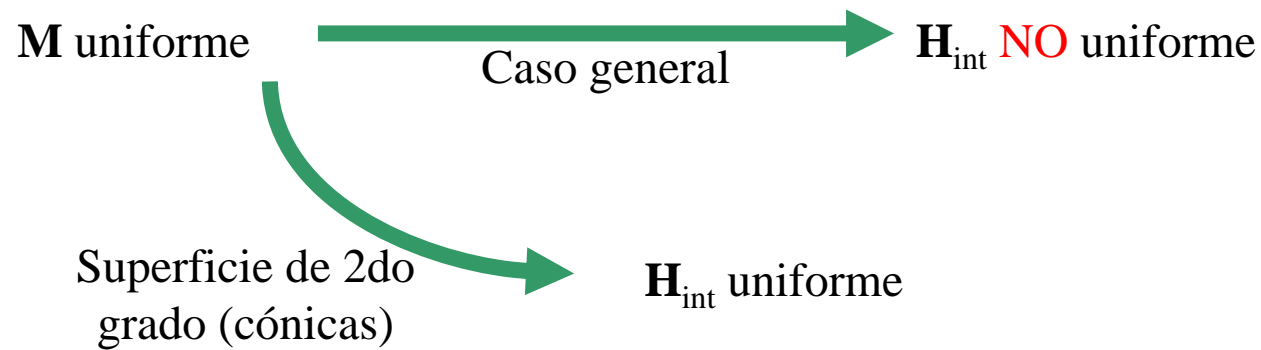
$$\vec{M} = M_S \check{x} \Rightarrow N_x = 1/2$$

$$\vec{M} = M_S \check{y} \Rightarrow N_y = 1/2$$

$$\vec{M} = M_S \check{z} \Rightarrow N_z = 0$$

$$N_x + N_y + N_z = 1$$

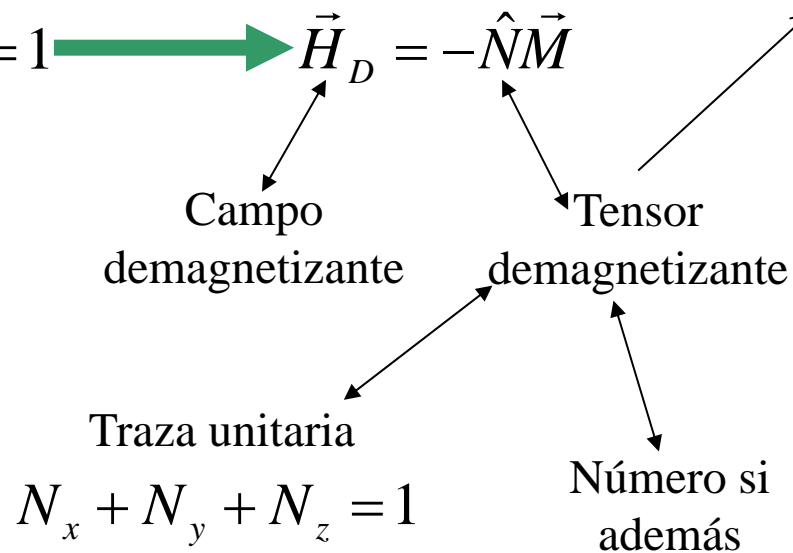
# Otros cuerpos magnetizados uniformemente



elipsoide

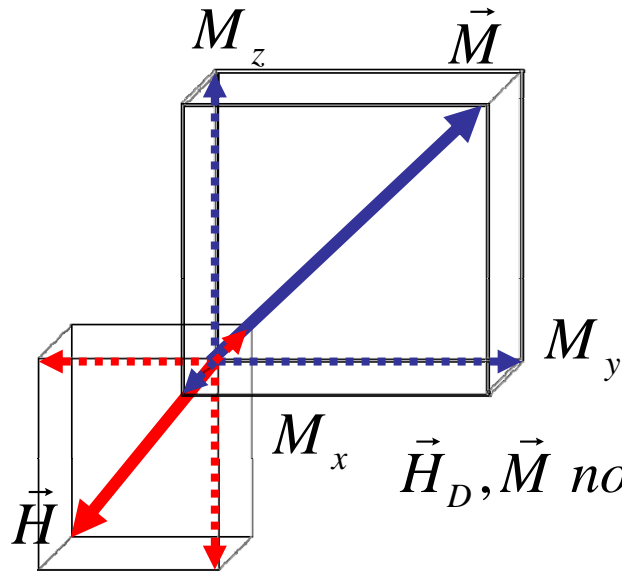
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$\mathbf{M}$  uniforme



Diagonal si los ejes de coordenadas coinciden con los del elipsoide

$$\vec{M} = \begin{cases} M_s \vec{i} \\ M_s \vec{j} \\ M_s \vec{k} \end{cases}$$



$$\vec{H}_D = -\hat{N}\vec{M} \quad \Rightarrow$$

$$H_x = -N_x M_x$$

$$H_y = -N_y M_y$$

$$H_z = -N_z M_z$$

$$N_x + N_y + N_z = 1$$

$\vec{H}_D, \vec{M}$  no colineales

$M_i, H_i, unif.$

$$E_M = -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV = -\frac{\mu_0}{2} (M_x H_x + M_y H_y + M_z H_z) V$$

$$E_M = \frac{\mu_0}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) V$$

$$E_M^{min} = 0, \text{ para } M = 0$$

## Superficies no cuadráticas

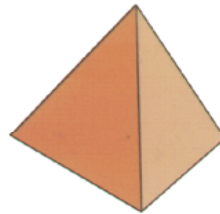
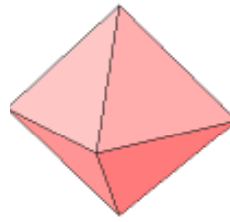
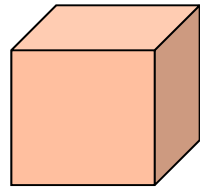
$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

Válido también para cuerpos con superficies no cuadráticas: cubos, prismas, cilindros, octaedros, etc.

(teorema de Brown-Morrish)

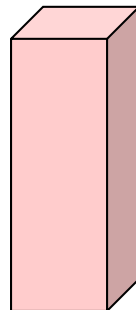
### Casos particulares

Cubo,  
octaedro,  
tetraedro



$$N_x = N_y = N_z = 1/3$$

Prisma  
regular,  
cilindro

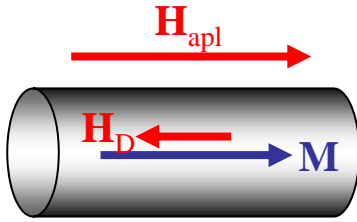


$$N_x = N_y \neq N_z$$

Caso límite

$$N_x = N_y = \frac{1}{2}; \quad N_z = 0$$

## campo efectivo



$$H_D = -NM$$

$$H_{ef} = H_{apl} + H_D = H_{apl} - NM$$

Cuando se grafica  $M$  vs.  $H$  debería usarse como abscisa el  $H_{ef}$

$$M \text{ vs } H_{ef} = H_{apl} - NM$$

$$\text{Si } M \ll M_S \implies M = \chi H_{ef}$$

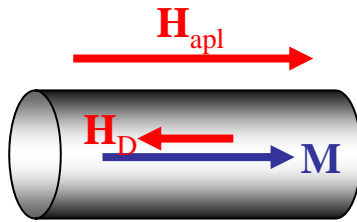
$$H_{ef} = H_{apl} - N\chi H_{ef} \implies H_{ef} = \frac{H_{apl}}{1 + N\chi} \implies M = \frac{\chi}{1 + N\chi} H_{apl}$$

$$\text{Cuando } \chi \ll 1 \implies M \approx \chi H_{apl} \quad \text{Independientemente de } N$$

$$\text{Cuando } \frac{\chi \gg 1}{N \sim 1} \implies M \approx \frac{1}{N} H_{apl} \quad \text{Fuertemente dependiente de } N$$

$\chi_{aparente}$

campo de saturación



$$H_{ef}^S = H_{apl} - NM_S$$

Ejemplo, Ni

$$M_S \approx 4.8 \times 10^5 \text{ A/m} \approx 0.6 \text{ Tesla}$$

$$H_{apl} - NM_S > H_S^{Ni} \xrightarrow{N \approx 1} H_{apl} > 0.6 \text{ Tesla} \quad H_S^{Ni} \approx 0.01 - 0.03 \text{ Tesla}$$



# factores demagnetizantes

## Cálculos en prismas

### Demagnetizing factors for rectangular ferromagnetic prisms

Amikam Aharoni<sup>a)</sup>

*Department of Electronics, Weizmann Institute of Science, 76100 Rehovoth, Israel*

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15 MARCH 1998

TABLE I. The demagnetizing factor,  $D_z^s$ , of a prolate spheroid and the magnetometric demagnetizing factor,  $D_z^p$ , of a square prism, for an aspect ratio,  $p$ .

$p$	$D_z^s$	$D_z^p$
2.0	0.17356	0.19832
3.0	0.10871	0.14036
4.0	0.075407	0.10845
5.0	0.055821	0.088316
6.0	0.043230	0.074466
7.0	0.034609	0.064363
8.0	0.028421	0.056670
9.0	0.023816	0.050617
10.0	0.020286	0.045731
11.0	0.017515	0.041705

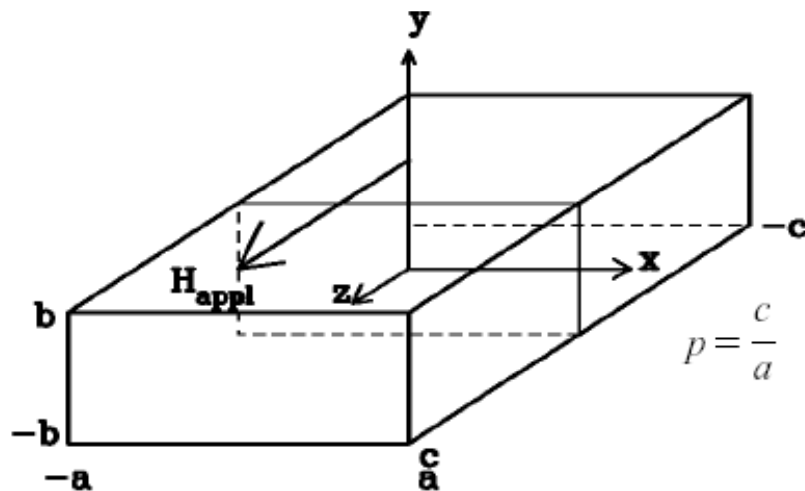


FIG. 1. The coordinate system used in the calculations. Its origin is at the center of the rectangular prism. The field  $H_{\text{appl}}$  is applied along the  $z$  axis.

# Factores demagnetizantes– referencias

Fórmulas, tablas y gráficos de factores demagnetizantes, Chen et al. IEEE Trans. Magnetics **27**, 3601-19 (1991)

Campo demagnetizante y medidas magnéticas, J.A. Brug y W.P. Wolf, J.Appl.Phys. **57**, 4685-701 (1985)

Cálculo de factores demagnetizantes,  
<http://magnet.atp.tuwien.ac.at/dittrich/?http://magnet.atp.tuwien.ac.at/dittrich/content/tools/magnetostatics/streufeld.htm>

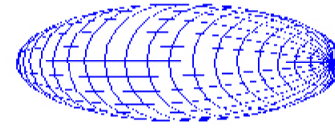
# Anisotropía de forma: NP elipsoidales

Elipsoide prolado u oblado

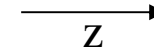
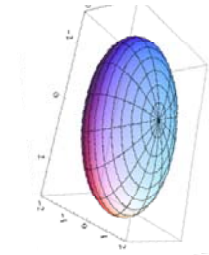
$$a = b < c \Rightarrow N_x = N_y > N_z$$

$$a = b > c \Rightarrow N_x = N_y < N_z$$

prolado



oblado



$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) = \frac{\mu_0 V}{2} (N_x (M_x^2 + M_y^2) + N_z M_z^2)$$

$$M_S^2 = M_x^2 + M_y^2 + M_z^2$$

$$E_M = \frac{\mu_0 V}{2} (N_z - N_x) M_z^2 + cte = \frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \cos^2 \theta + cte$$

$$E_M = -\frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \sin^2 \theta + cte = K_S V \sin^2 \theta + cte$$

$$E_M = K_{ME} V \sin^2 \theta$$

$$K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2$$

Ejemplo: elipsoide prolado largo de magnetita

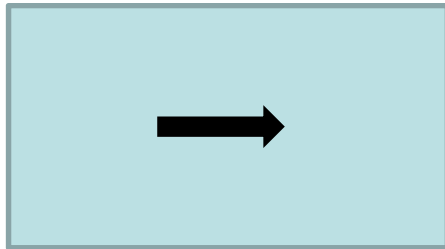
$$K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2; \quad N_x \approx 1/2, N_z \approx 0$$

$$M_S \approx 4.5 \times 10^5 \text{ A/m}$$

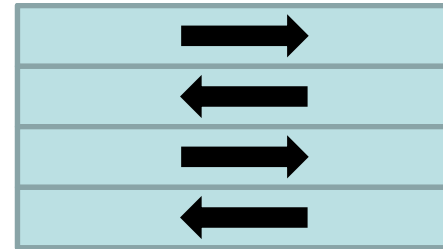
$$K_{ME} = \frac{4\pi \times 10^{-7}}{2} \frac{1}{2} (4.5 \times 10^5)^2 \text{ J/m}^3 \approx 6.4 \times 10^4 \text{ J/m}^3$$

# Energía magnetostática - Origen de los dominios

1 dominio



n dominios

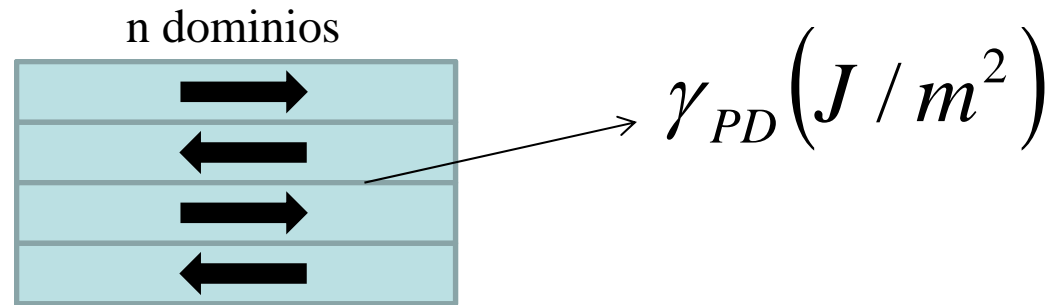


$$E_M(n) = C(n)\mu_0 M_S^2$$

$$\frac{E_M(n)}{E_M(1)} \approx \frac{1}{n}$$

Regla  
aproximada

## Energía de pared de dominios



$$E_{PD} \approx S_{PD}(n) \gamma_{PD} (J / m^2)$$

# Número de dominios en equilibrio



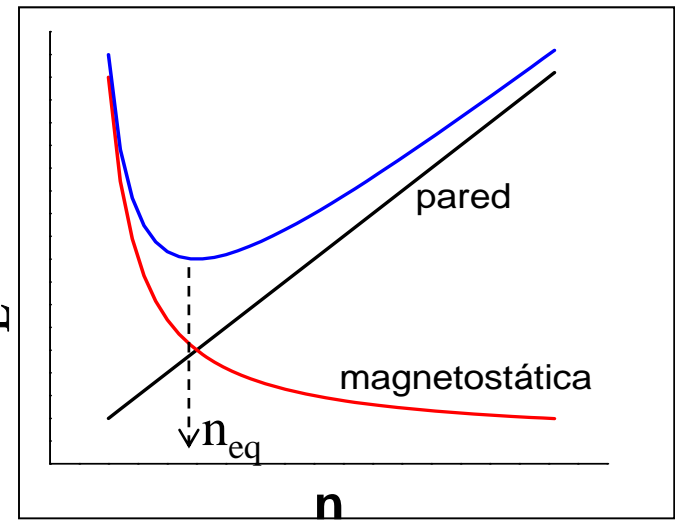
$$E^{multi} \approx \frac{E_M^{mono}}{n} + S_{PD}(n)\gamma_{PD}$$

Energía de pared por unidad de área

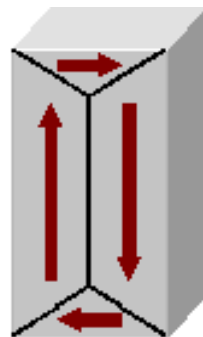
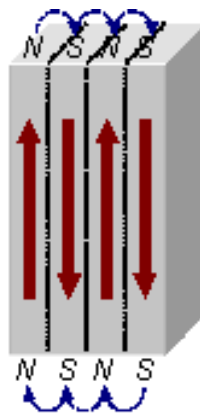
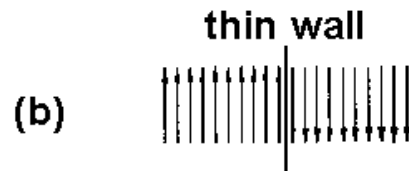
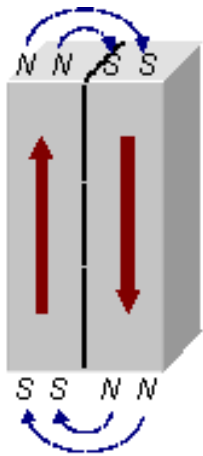
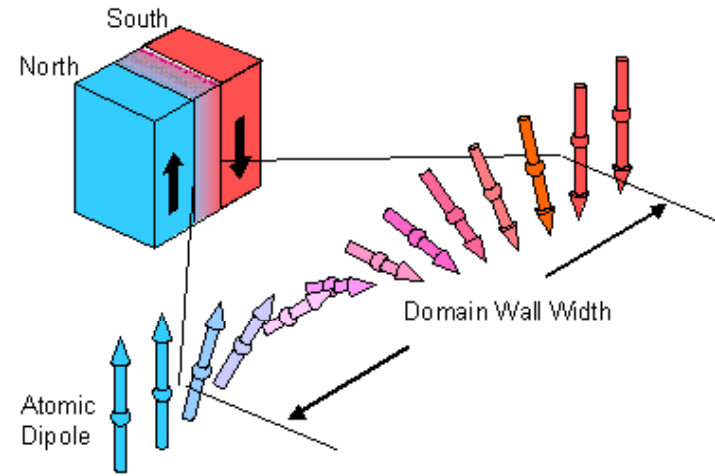
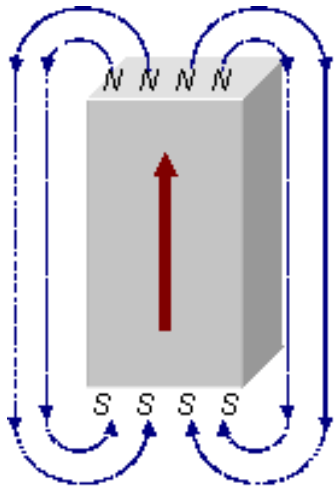
E magnetostática decrece

E pared dominios crece

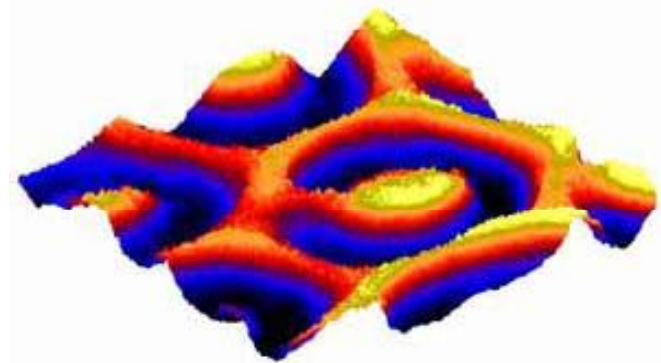
Nro dominios en equilibrio



# Dominios y paredes de dominio



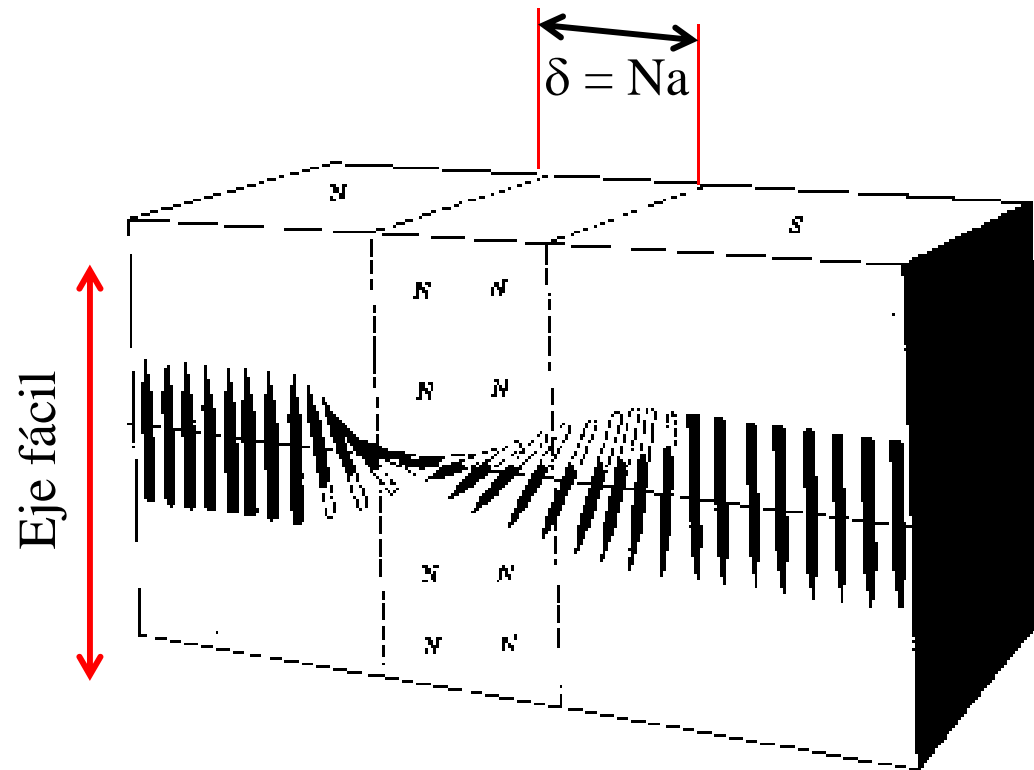
**MFM**

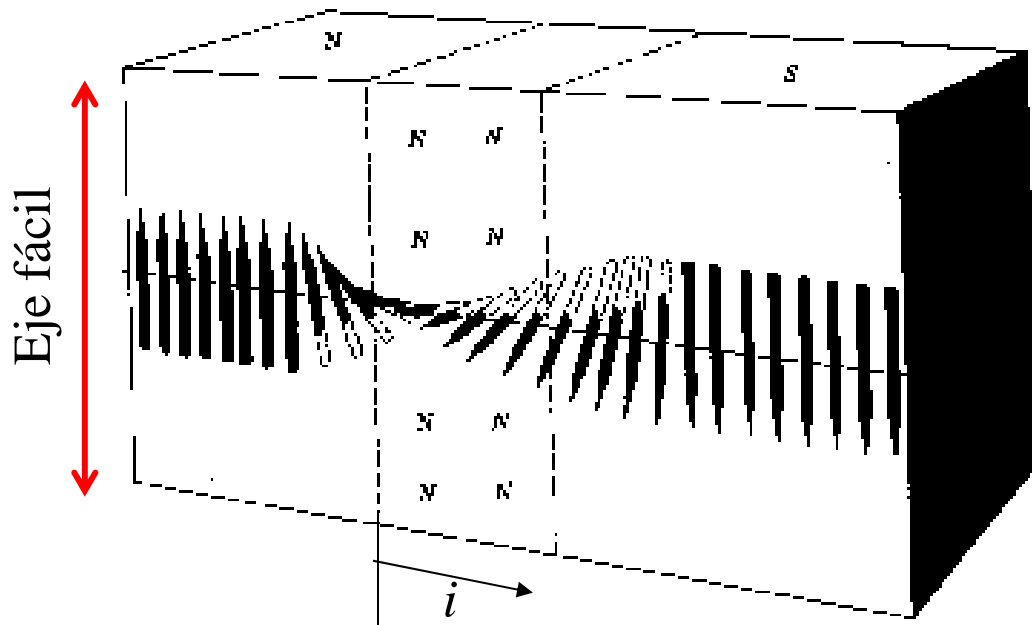


Pseudo-3d MFM image of a  $(\text{YSmLaCa})_3(\text{FeGe})_5\text{O}_{12}$  magnetic thin film garnet,  $4.5 \times 4.5 \mu\text{m}^2$ , domain walls appear dark;



# Pared de Bloch de 180°





Balance energético entre  
anisotropía e intercambio



$$E_K = a^3 \sum_1^N K \text{sen}^2 \theta_i$$

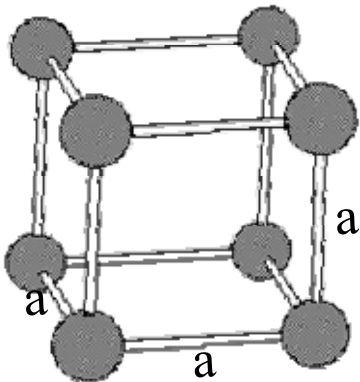
Para un prisma de  $N$  celdas (base  $a^2$ )  
 $a$ : parámetro de la celda cúbica

$$\theta_i = \pi i / N$$

$$E_K = a^3 \sum_i K \text{sen}^2 \theta_i \Delta i \quad \leftarrow \quad \Delta i = \frac{N \Delta \theta_i}{\pi} = 1$$



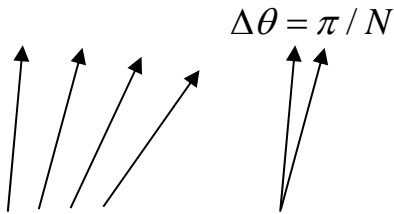
$$E_K = \frac{KNa^3}{\pi} \sum_i \text{sen}^2 \theta_i \Delta \theta \approx \frac{KNa^3}{\pi} \int_0^\pi \text{sen}^2 \theta_i d\theta = \frac{KNa^3}{2}$$



Por unidad de área  
de pared:  $\epsilon_K \approx \frac{KNa}{2}$

intercambio

$$E_J = -2NJs^2 \cos \Delta\theta = -2NJs^2 \cos(\pi / N), \quad \pi / N \ll 1$$



$$E_J = -2NJs^2 \left(1 - \frac{\pi^2}{2N^2}\right)$$

$$\varepsilon_J = -\frac{2NJs^2}{a^2} \left(1 - \frac{\pi^2}{2N^2}\right)$$

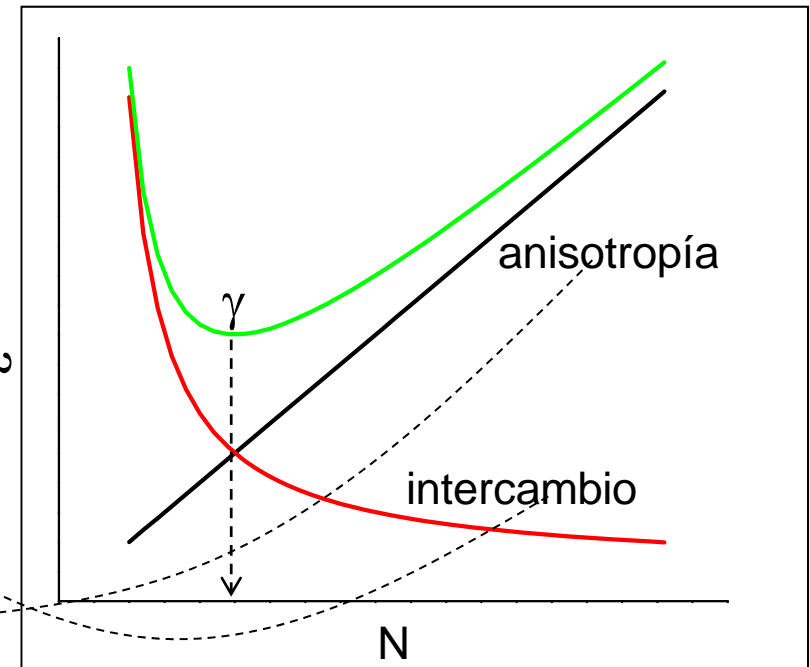
Energía de intercambio por unidad de área de pared:

Relativa al estado fundamental:

$$\boxed{\varepsilon_J = -\frac{2NJs^2}{a^2}} \Rightarrow \Delta\varepsilon_J = \frac{\pi^2 Js^2}{Na^2}$$

Energía por unidad de área de pared

$$\gamma = \Delta\varepsilon_K + \Delta\varepsilon_J = \frac{KNa}{2} + \frac{\pi^2 Js^2}{Na^2}$$



Optimización energía por unidad de área de pared

$$\gamma = \Delta\varepsilon_K + \Delta\varepsilon_J = \frac{KNa}{2} + \frac{\pi^2 Js^2}{Na^2}$$

$$d\gamma / dN = \frac{Ka}{2} - \frac{\pi^2 Js^2}{N^2 a^2} = 0$$



$$N_{eq} = \pi \sqrt{\frac{2Js^2}{Ka^3}}$$

Ancho de la pared  $\delta_{eq} = N_{eq} a = \pi \sqrt{\frac{2Js^2}{Ka}} = \pi \sqrt{\frac{2A}{K}}$

$A \approx Js^2 / a$   
Cte de stiffness

$$\gamma_{eq} = 2\pi \sqrt{2KA}$$

$$10^{-12} J / m \leq A \leq 10^{-11} J / m$$

$$10^3 J / m^3 \leq K \leq 10^6 J / m^3$$

$$A = 10^{-11} J / m$$

$$K = 10^3 J / m^3 \Rightarrow \delta_{eq} = 444 \text{ nm}$$

$$K = 10^5 J / m^3 \Rightarrow \delta_{eq} = 44.4 \text{ nm}$$

*Fin módulo*

FIN

Casi...

# Examen

## Coloquio con público

- Tema de interés del disertante (abordando los temas del curso: intercambio, anisotropía, magnetostática, etc.)
- Tema asignado (idem)