Selfconsistent domain theory in softferromagnetic media. II. Basic domain structures in thinfilm objects

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Self-consistent domain theory in soft-ferromagnetic media. II. Basic domain structures in thin-film objects

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A construction method for determining the domain structure in ideally soft-ferromagnetic cylindrical objects with plane-parallel top and bottom surfaces of arbitrary shape is presented. The self-consistent theory is confined to two-dimensional solenoidal dipole distributions in which the dipoles are parallel to the top and bottom surfaces. It is proved that the basic domain structure is uniquely defined in simply connected objects, while an extra criterion has to be added in order to guarantee the uniqueness in the multiply connected ones. The treatment is based on differential geometrical principles. The object edge is partitioned into segments, in which each segment is situated in between two adjacent edge points where the radii of curvature of convex edge segments are locally minimal. To each edge segment, a region is attributed, in which **M** is uniquely specified by the course of **M** along that edge segment. In the cross section of regions corresponding to different edge segments, domain walls provide an adequate separation of the dipole distribution imposed by these segments. The extremities of these domain walls are found in the singular points of the evolute corresponding to the extremities of the edge segments and in the points where a number of walls meet. It is proved that the basic domain structure is the locus of centers of all circles inside the object that touch the object edge at at least two points. A number of experimentally observed basic structures are given, and the relevance of the definition of basic structures in multiply connected objects is examined.

I. INTRODUCTION

In paper I,[1] we considered the magnetization distribution at a zero external field in an ideally soft-magnetic plane-parallel object with an elliptical cross section. In this paper, we shall generalize our theoretical approach in order to cover the so-called basic domain structures in thin-film objects with arbitrary shapes. The basic domain structure is, in general, the simplest two-dimensional domain configuration in a given object, while its counterpart—the composite structure—exhibits a much greater complexity and variety. The latter category will come up for discussion in a forthcoming paper.

Let us recapitulate the key ideas and equations of paper I[1]: In ideally soft-magnetic thin-film objects,the dipole distribution is solenoidal at a zero external field and is governed by the following equations:

$$-\xi \frac{\partial \xi}{\partial x} + \frac{\partial \xi}{\partial y} = 0, \quad \xi = \frac{M_y}{M_x},$$

and

$$M_z \equiv 0 \quad \text{in } V, \tag{1a}$$

where M_x and M_y are the lateral magnetization components along the axis of a Cartesian-coordinate system. The boundary condition at the object edge S is given by

$$\mathbf{M}\cdot\mathbf{n} = 0 \quad \text{on } S, \tag{1b}$$

where $\mathbf{n}$ is the outwardly directed unit vector perpendicular to the edge. The characteristic base curves $\mathbf{r}'(t,s)$ of Eq. (1) are straight lines perpendicular to the edge and satisfy[1]

$$\mathbf{r}'(t,s) = \mathbf{r}(s) + \boldsymbol{\tau}(s)t, \tag{2}$$

where $\mathbf{r}(s)$ is the parametric vector representation of the edge, $\boldsymbol{\tau}(s)$ is the inwardly directed unit vector perpendicular to the edge at $\mathbf{r}(s)$, while t is a position parameter along the base curve. Moreover, it was proved[1] that $\mathbf{M}\cdot\boldsymbol{\tau}(s) = 0$ at the base curve, so that an ambiguity in the **M** direction occurs at the intersection of two base curves that originate in two edge points whose unit vectors **n** are not parallel. Hence, the range of definition of the characteristics will inevitably be confined to a limited range adjacent to the edge point in which they originate. Bear in mind that the characteristics of edge points with infinitesimal mutual distance intersect each other in the center of the local radius of curvature of the corresponding edge segment. Therefore, the truncation of the characteristics should, in any case, be within the region in between the locus of centers of the radii of curvature of a specific edge segment and this segment itself. This locus $\mathbf{r}''$ is called the evolute of the edge segment and is given by

$$\mathbf{r}''(s) = \mathbf{r}(s) + R_c(s)\mathbf{n}_c(s), \tag{3a}$$

where $\mathbf{n}_c(s)$ is the unit vector perpendicular to the edge at point $\mathbf{r}(s)$ that points towards the concave side of the edge, while the radius of curvature $R_c(s)$ is given by

$$\mathbb{R}^{-1} = \frac{\mathbf{n}_c}{R_c} = \frac{d\,\mathbb{T}(s)}{ds} \Big/ \left(\frac{d\mathbf{r}}{ds}\cdot\frac{d\mathbf{r}}{ds}\right)^{1/2}, \tag{3b}$$

where

$$\mathbb{T}(s) = \frac{d\mathbf{r}(s)}{ds} \Big/ \left(\frac{d\mathbf{r}}{ds}\cdot\frac{d\mathbf{r}}{ds}\right)^{1/2}, \tag{3c}$$

in which $\mathbb{R}^{-1}$ is called the vector of curvature. From the above concepts of differential geometry, we shall extract a general methodology for determining domain structures in arbitrary thin-film objects. In this connection, the points of the evolute where R_c of convex parts of the edge has a local minimum will play an essential part, as we shall see. These points of the evolute are the closest to the object edge and each such point constitutes one of the extremities of two different branches of the evolute.

II. BASIC DOMAIN STRUCTURES IN SIMPLY CONNECTED OBJECTS

In this section, we shall confine ourselves to simply connected thin-film objects. These are objects having the property that any closed curve within the object encloses a region that completely belongs to the object. Within this category, we shall distinguish the so-called convex objects and objects with concave edge segments. To start with, we shall treat the convex objects, while the procedure developed there will be set forth in Sec. II B on objects with concave edge segments.

A. Convex objects

Convex objects are defined by the following requirement: Any straight line in between any pair of points of the object is completely situated within the object. Clearly, a convex object is always simply connected. For these convex objects, we shall present a construction method by which the basis domain structures can be derived. Ultimately, this method will be united into one single criterion governing the positions of the domain walls.

In the introduction, we have emphasized the necessity for defining regions adjacent to the edge or certain segments of the edge in which $\mathbf{M}$ is uniquely specified. This implies that the sphere of influence of each characteristic has to be limited to a finite distance adjacent to the edge point in which it originates. For that purpose, the evolute is a natural boundary, so that the distance parameter $t(s)$ in Eq. (2c) is limited to

$$|t(s)| < R_c(s) . \quad (4)$$

Let us have a closer look at the evolute. The radius of curvature $R_c(s)$ exhibits an alternating course with local maxima and minima as a function of s. In between each local minimum and one of its adjacent maxima, one branch of the evolute with a continuous directional derivative extends itself. A discontinuity in the derivative is present at the extremities of these branches where two neighboring branches link.

Let us introduce a partitioning of the edge based on the edge points $\mathbf{r}(s_i)$ with a local minimum in the radius of curvature. The subscript i of the position parameter s is an index that successively increases by one when tracing the perimeter in a counterclockwise direction. When there are n of these points at a specific edge, it applies that $\mathbf{r}(s_{n+1}) = \mathbf{r}(s_1)$. Now, we replace the parameter s by a set of n new parameters $s^1, s^2, ..., s^n$, where s^i defines the edge points between points $\mathbf{r}(s_i)$ and $\mathbf{r}(s_{i+1})$.

The evolute of edge segment (i) consists of two branches, being separated by $\mathbf{r}''(s^i)$ corresponding to the local maximum of R_c between $\mathbf{r}''(s_i)$ and $\mathbf{r}''(s_{i+1})$. The characteristics touch the evolute and are its generators (see Ref. 2, p. 314). It can be seen that characteristics of a specific segment do not intersect each other when they are truncated in accordance with Eq. (4). In other words, $\mathbf{M}$ is uniquely specified, apart from an angle of 180°, by the characteristics of segment (i) in the region Ω_i, located between segment (i), its evolute and the radii between $\mathbf{r}(s_i)$ and $\mathbf{r}''(s_i)$ and between $\mathbf{r}(s_{i+1})$ and $\mathbf{r}''(s_{i+1})$. However, a different situation presents itself as soon as we consider a truncated characteristic from another edge segment, let us say (s_{i-1}). This situation is the subject of the next section.

1. Dipole distribution near edge points with a local minimum in R_c

The shape of the edge near such a local minimum at $x = 0$ can be expanded in the following Taylor series:

$$y = f(x) = ax^2 + bx^3 + \cdots, \quad a > 0 \quad (5)$$

in which, for convenience, the y axis of the Cartesian-coordinate system has been chosen along $\tau(s)$ at an edge point with a local minimum in R_c. For sufficiently small x, Eq. (5) may be approximated by

$$y = ax^2 . \quad (6)$$

Instead of Eq. (3), it is more convenient to employ the following representation of the evolute of the curve $y = f(x)$ (see Ref. 3, p. 415):

$$x'' = x \left\{ \frac{df}{dx} \left[1 + \left(\frac{df}{dx} \right)^2 \right] \Big/ \frac{d^2 f}{dx^2} \right\} , \quad (7a)$$

$$y'' = y + \left[1 + \left(\frac{df}{dx} \right)^2 \Big/ \frac{d^2 f}{dx^2} \right] , \quad (7b)$$

from which we obtain for the parabola (6):

$$\left(y'' - \frac{1}{2a} \right)^3 = \left(\frac{3}{4a} \right)^3 (2ax'')^2 . \quad (7c)$$

This evolute is depicted in Fig. 1 and forms the boundary between a shaded and an unshaded region within the parabola. In the unshaded region each point is located at one characteristic only, while the remaining points are situated on two base curves, when these are truncated according to (4), and, if not, on three base curves. As a consequence, the

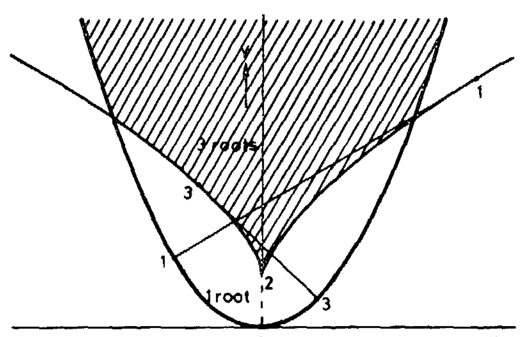

FIG. 1. The evolute of a parabola and the number of nontruncated characteristic base curves that intersect in the various regions.

sphere of influence of the characteristics has to be reduced still further in order to prevent ambiguities in the dipole direction from occurring in the shaded area. These ambiguities occur as soon as we consider the continuation of the dipole distribution from the segments at both sides from $x = 0$. As discussed previously,[1] we have to drop the requirement of continuity to the dipole distribution and have to allow domain walls to separate the dipole distributions imposed by the edge segments on both sides of $x = 0$. It is self-evident that such a domain wall has an extremity at the singularity in the evolute $\mathbf{r}''(s_2)$. The above image has to be quantitatively corrected in the case of the general curve (5), implying that the shape of the evolute has to be modified; however, the general conclusion that a domain wall has one of its extremities in the singular point of the evolute still holds. In the next section, we shall prove another property of domain walls, which greatly simplifies the prediction of the domain structure.

2. The position of domain walls

Let us consider the domain wall that separates two distributions imposed by two arbitrary edge segments at which two position parameters s^1 and s^2 are defined and at which **M** is as indicated in Fig. 2. The characteristics of two edge points $\mathbf{r}(s_2^1)$ and $\mathbf{r}(s_2^2)$ intersect at point P in which $\mathbf{r}'(t^1,s_2^1) = \mathbf{r}'(t^2,s_2^2)$. At a possible domain wall in this point, the bisector relation[4] has to be satisfied, so that this domain wall in P is parallel to the bisector of both tangents to the edge at points $\mathbf{r}(s_2^2)$ and $\mathbf{r}(s_2^1)$ (see the lines indicated by C and D in Fig. 2. Let us define a function d to determine the difference in the distance between a point at the domain wall and the corresponding points on both edge segments. It follows from Eq. (2) that

$$d(s_2^1,s_2^2) = t(s_2^2) - t(s_2^1) .$$

Subsequently, we define a position parameter u along the domain wall through P, which is zero at P. The distance between point Q at a distance δu from P along the wall and point Q' at the same distance along the straight line B is of the order $(\delta u)^2$ for infinitesimal δu. It is obvious that the difference in the distance between points on the line B to both tangents C and D is constant. Moreover, it can be seen from Fig. 2 that the difference in the distance from Q to the edge segment with parameter s^2 and from Q' to the line D is of the order $(\delta u)^2$, so that

$$d(Q) - d(P) = O[(\delta u)^2] ,$$

which implies that

$$d\, d(u)/du = 0 . \quad (8)$$

In other words, the distance function $d(u)$ is constant along a domain wall.

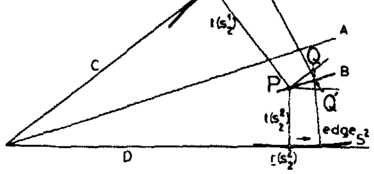

FIG. 2. The course of the distance function $d(s_2^1,s_2^2)$ along a domain wall which separates the magnetization distributions imposed by two edge segments denoted by the parameters s^1 and s^2.

Let us return to the domain wall of the previous section which has one of its extremities in the singular point of the evolute. The characteristics from the segments at both sides of $\mathbf{r}(s_2)$ through this point $\mathbf{r}''(s_2)$ in Fig. 1 coincide so that the distance function $d(u)$ is zero here. Therefore, it can be concluded that each point at a domain wall with one extremity in point $\mathbf{r}''(s_i)$ has equal distances to the edge segments with the position parameters s_{i-1} and s_i.

3. Construction of basic domain structures

After having established two properties of the domain-wall positions, we shall pass onto the construction of the basic domain structures in convex objects and onto a description of domain structures in terms of the locus of centers of circles that touch the edge. The absence of edge clusters[5,6] is a prerequisite for the occurrence of the basic domain configurations. A sharp corner in the edge will be considered as a degenerated rounded corner, in which the singularity in the evolute coincides with the corner itself. The following steps in the construction of a basic structure can be distinguished.

(1) Our starting point is the decomposition of the edge into j segments with position parameters s^i $(i = 1,...,j)$, as discussed in Section II A. The segment (i) is bounded by the adjacent points $\mathbf{r}(s_i)$ and $\mathbf{r}(s_{i+1})$ at which the radius of curvature is locally minimal. To each segment (i), a region Ω^i is attributed in which **M** is, apart from an angle of 180°, uniquely specified by the characteristics originating in that segment, so that one unique value of s^i corresponds to each point P in Ω^i.

(2) In the cross section $\Omega^k \cap \Omega^{k+1}$ of each pair of adjacent segments (k) and $(k+1)$, a vector field **W** can be defined [see Ref. 1, Eq. (20)] from which a domain wall can be derived by means of the equation

$$\frac{dy}{dx} = \frac{W_y}{W_x} ,$$

which starts at the point $\mathbf{r}''(s_{k+1})$, which, for convenience, shall be denoted by $Q'_{k,k+1}$. We shall refer to these points $Q'_{k,k+1}$ as points of the first order. This domain wall can easily be constructed since it is the locus of the centers of all circles that touch both segments (k) and $(k+1)$. In the appendix, it is shown that circles that touch both segments (k) and $(k+1)$ do not intersect these two segments at any point. The above procedure is repeated for all combinations of adjacent segments, so that we arrive at at most j domain walls.

(3) Subsequently, we consider the points of intersection of adjacent domain walls that are located within the object. The point of intersection of the domain walls with extremities in $Q'_{k,k+1}$ and $Q'_{k+1,k+2}$ is denoted by $Q_{k,k+2}$, in which the subscripts (k) and $(k+2)$ refer to both exterior edge segments of the segments involved, namely, (k), $(k+1)$, and $(k+2)$. Note that the circle with center $Q_{k,k+2}$ that touches the segments (k) and $(k+2)$ also touches the interior segment $(k+1)$, so that $Q_{k,k+2}$ is at equal distances to three segments. The intersection of two adjacent domain walls with extremities in points of the first order will be called a point of the second order. Bear in mind

that $(k+2)$ is not always smaller than $(j+1)$, so that the subscripts of the intersections $Q_{k,l}$ of the second order obey the following scheme:

$$l-k=2 \quad \text{when } 1 \leqslant k < l \leqslant j,$$

and

$$l-k=2-j \quad \text{when } l=1,2. \tag{9}$$

It should be noted that the lengths of the domain walls are confined to the area of intersection of adjacent Ω's. Therefore, adjacent domain walls do not always intersect, so that the number of second-order points may be smaller than j [see Fig. 3(a)].

(4) In the next step, we select the second-order point, let us say $Q_{k,l}$ $(l=k+2)$, [$Q_{3,5}$ in Fig. 3(a)], that has the shortest distance to the edge. We constitute a new collection of first-order points. Point $Q_{k,l}$ is added to this collection, and is now denoted by $Q'_{k,l}$, while the points $Q'_{k,k+1}$ and $Q'_{l-1,l}$ are removed from the original first-order collection. Note that the circles touching at segments (k), $(k+1)$, and $(k+2)$, whose centers constitute the domain walls connecting the points $Q'_{k,k+1}$ and $Q'_{k+1,k+2}$ with the new first-order point $Q'_{k,k+2}$, do not intersect any edge segment of the object. Moreover, these domain walls are constituted by the centers of all circles that touch simultaneously at segment $(k+1)$ and either segment (k) or $(k+2)$.

(5) Next, we construct a domain wall with an extremity at the new first-order point $Q'_{k,l}$ [in which l is given by (9)]. Thus, the domain walls with extremities at $Q'_{k,k+1}$ and $Q'_{l-1,l}$ are replaced by one domain wall, whose course is determined by segments (k) and (l). Effectively, we have reduced the number of edge segments by one because the segment $(k+1)$ is hidden by the domain walls with extremities at $Q'_{k,k+1}$ and $Q'_{k+1,k+2}$. Note that the domain wall is the locus of centers of all circles that touch at both segments (k) and (l) and that none of these circles intersects the edge segment $(k+1)$ (see the Appendix).

(6) Subsequently, we recompose the collection of second-order points by removing $Q_{k,l}$ $(l=k+2)$ and adding the points of intersections of the domain walls with extremities in $Q'_{k,l}$, $Q'_{k-1,k}$, and $Q'_{l,l+1}$ of the new collection of first-order points [see Fig. 3(b)]. Of course, these points, denoted by $Q_{k-1,l}$ and $Q_{k,l+1}$, need not always exist because of the finite length of the segments $(k-1)$, (k), (l), and $(l+1)$. The aforementioned transformations are summarized in Table I.

(7) We repeat the procedure given in steps (4)–(6) by searching for the second-order point of the new collection, let us say $Q_{l,n}$, having the shortest distance to the corresponding edge segments. In general, n is not equal to $(l+2)$ while the subscript m of the first-order points involved $Q'_{l,m}$ and $Q'_{m,n}$ is within the set $[(l+1), (l+2),...(n-2),(n-1)]$ (for simplicity, we have assumed that $1 \leqslant l < n \leqslant j$). In previous steps, the segments $(l+1)$ to $(m-1)$ and $(m+1)$ to $(n-1)$ have efficaciously been screened by domain walls. Note that the circle with center $Q_{l,n}$ that touches segments (l), (m), and (n) is completely situated within the object [see the above step (5)]. Subsequently, $Q_{l,n}$ is added to and $Q'_{l,m}$ and $Q'_{m,n}$ are removed from the collection of first-order points. A domain wall starting in the new $Q'_{l,n}$ is constructed by determining the locus of centers of all circles that touch at both segments (l) and (n). Note again that none of these circles intersects any segment $(l+1)$ to $(n-1)$ (see the Appendix).

(8) By repeating the above procedure, the number of effective edge segments governing the magnetization in the region where the course of the domain walls is not yet determined is systematically reduced. Ultimately, we arrive at two first-order points. The ultimate domain wall is simply the locus of centers of circles that touch at both segments. Again, these circles do not intersect any edge segment. Therefore, finally, we can conclude that the basic domain configuration is the locus of all circles that touch the edge at at least two points and which do not intersect the object's edge at any point.

Remark: Sometimes, adjacent domain walls do not intersect because the locally minimal radius of curvature, let us say of the point $\mathbf{r}(s_3)$ [see Fig. 3(b)], is larger than the distance of its adjacent domain wall to $\mathbf{r}(s_3)$. In this case, a

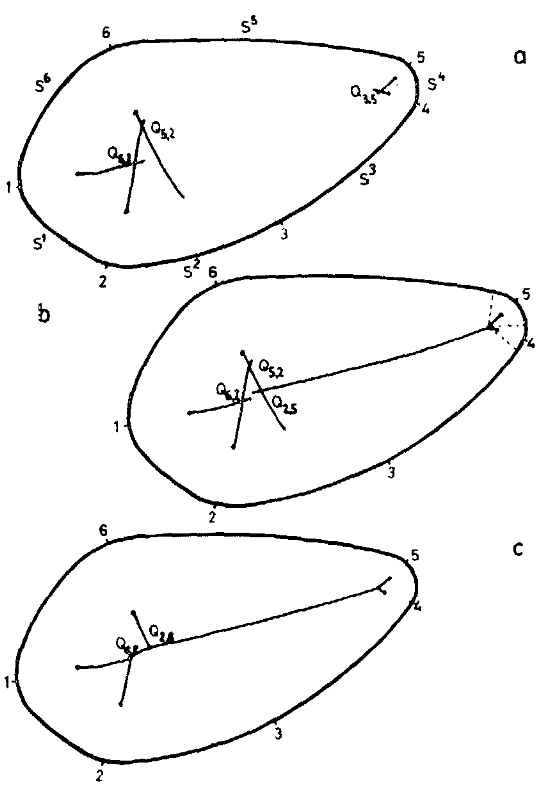

FIG. 3. The construction of the basic domain structure in a simply convex object.

TABLE I. Transformation of points of first and second order when reducing the number of effective edge segments by one.

	Original	Replaced by
Segments	(k), $(k+1)$, $(k+2)$	(k) and $(k+2)$
Points of the first order	$Q'_{k,k+1}$ and $Q'_{k+1,k+2}$	$Q'_{k,k+2}$
Points of the second order	$Q_{k-1,k+1}$, $Q_{k,k+2}$, and $Q_{k+1,k+3}$	$Q_{k-1,k+2}$ and $Q_{k,k+3}$

domain wall originating in another first-order point will smoothly connect with the latter domain wall [see Fig. 3(c)].

Remark: In principle, we can also end up with three first-order points and with three segments that are usually not next to each other. It can easily be seen that the three domain walls departing from these first-order points intersect at one point.

B. Objects with concave edge segments

Up to now, we have confined ourselves to the domain structure in convex specimens, e.g., objects in which the edge observed from the outside is convex at all points. In order to cover the domain structure in general simply connected geometries, we have to pay attention to the general characteristics of the concave segments.

In Fig. 4, we have depicted such a concave segment together with the adjacent convex segments at both sides up to the points $\mathbf{r}(s_i)$ and $\mathbf{r}(s_{i+1})$ where the radii of curvature have local minima. As we did previously, we shall define a region Ω^i in which each point P is intersected by only one member of the characteristic base lines that originate in the segment between $\mathbf{r}(s_i)$ and $\mathbf{r}(s_{i+1})$ when these are truncated in accordance with Eq. (4). The evolute of the segment will be our starting point. Four branches can be recognized [see Fig. 4(a)]. The radii of curvature are infinite at two points $\mathbf{r}(s_1^i)$ and $\mathbf{r}(s_2^i)$, being those points on the smooth edge with a continous derivative d^2f/dx^2 [see Eq. 7(a)], where the concave part twins into its convex neighbors. The branches approach asymptotically to the characteristics through these points $\mathbf{r}(s_1^i)$ and $\mathbf{r}(s_2^i)$. The region Ω^i is bounded by the edge segment between $\mathbf{r}(s_i)$ and $\mathbf{r}(s_{i+1})$, the radii between the pair of points $\mathbf{r}(s_i)$ and $Q'_{i-1,i}$ and the pair $\mathbf{r}(s_{i+1})$ and $Q'_{i,i+1}$ and both outermost branches of the evolute. The area of Ω^i is infinite; however, it can easily be seen that one characteristic can be attached to each point P in Ω^i, when constraint (4) truncates the characteristics of the convex parts. A straight edge segment [see Fig. 4(b)] is an intermediate form of a convex and concave segment for which the above definition for Ω_i or the one of Sec. II A are equivalent.

Let us subsequently consider the real objects whose edges contain these concave sections. Again, we start by determining the positions along the convex parts of the edge in which the radii of curvature possess local minima. These points demarcate the segments of the subdivision of the edge. The basic domain configuration follows from the same procedures as developed in Sec. II A 3. for the convex objects. An illustration of this method is provided by Fig. 5.

The question remains of whether the basic structure in any simply connected object, thus also those with concave segments, is the locus of the centers of all circles inside the object that touch the edge at at least two points. The answer is affirmative because the construction of the basic structure in this section proceeds along the same lines as for convex objects, while the conclusions of the Appendix remain valid. On the other hand, this theorem is hardly a law of the Medes and Persians, as we shall discover in the next section on the multiply connected objects.

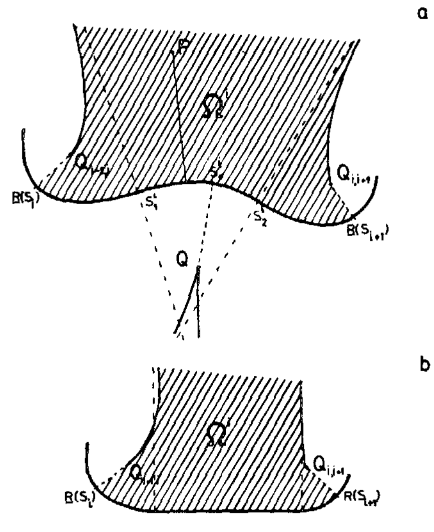

FIG. 4(a) The definition of Ω^i of an edge segment with a concave part. (b) The region Ω^i of an edge segment with a straight part.

III. BASIC DOMAIN STRUCTURES IN MULTIPLY CONNECTED MEDIA

In the previous sections, we have confined ourselves to simply connected media. For every closed curve that is completely situated within a simply connected object, it applies that every point of the region enclosed by this perimeter also belongs to the object. The present category of objects, in which "holes" are present, can be treated analogously to the simply connected ones; however, as we shall see, a few additional steps are required to incorporate them into our framework. As the starting point of our discussion, let us have a closer look at the very simple example provided by Fig. 6(a).

The object contains one hole with edge (2), in which no

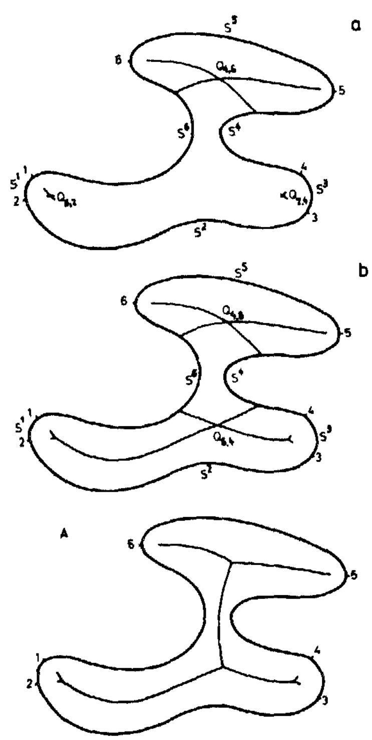

FIG. 5. The different phases in the construction of the basic domain configuration in a specimen with three concave segments.

magnetic dipoles are present. From the exterior edge (1), the dipole distribution extends itself into the interior of the object, while a different dipole configuration is imposed by the interior edge (2). In general, both configurations are incompatible and, therefore, should be separated by an adequate domain-wall configuration. In contradistinction to simply connected objects, it is not self-evident where the extremities of the separating walls are situated, because, in principle, the interior edge (2) need not contain a single convex segment. In order to construct the separating domain-wall structure between edges (1) and (2), we shall employ the concept of the involute (Ref. 3, pp. 417–8). These involutes of edge (1) are lines perpendicular to the family of characteristics that corresponds to the specific edge. The distance, measured along the characteristics, between each corresponding pair of points on the edge and a specific involute is a constant quantity for that involute. In some respect, each involute is "parallel" to the original edge; however, a distinctly visible alteration in the shape of the auxilary edge with respect to edge (1) occurs when the mutual distance between edge (1) and the auxiliary edge is larger than the smallest radius of curvature of a convex part of edge (1). In this case, a discontinuity in the outwardly directed unit vector **n** normal to this auxiliary edge occurs [compare edges (1) and (1″) in Fig. 6(b)]. Note that this point of discontinuity of the auxilary edge is situated on the domain wall that has its extremity in the corresponding center of the local minimum of the radius of curvature of edge (1).

Even so, we construct a family of auxiliary edges "parallel" to edge (2) by drawing closed curves in between edges (1) and (2) that intersect all characteristics corresponding to edge (2) at right angles [see Fig. 6(c)]. It should be observed that a corner at edge (2) is replaced by a circular segment [see Fig. 6(c)], so that this second family of auxiliary edges does not exhibit discontinuities in the direction of the unit vector **n**.

Now, the construction of the adequate domain-wall configuration proceeeds as follows. Take from each family of auxiliary edges one member such that these members touch each other, and thus have no point of intersection, while, in addition, the member corresponding to edge (1) encloses the one of the second family. It is obvious that an infinite number of these pairs can be selected. Subsequently, we construct a basic domain configuration in the region bounded by a given pair (1′) and (2′) of these auxiliary edges that corresponds to edges (1) and (2), respectively. Note that these auxiliary edges enclose a concave simply connected subregion of the object. Next, we construct the fragments of the basic domain structure imposed by edge (1), while the presence of the hole is neglected, in between edges (1) and (1′). Note that the latter fragments link up smoothly with the basic structure in between edges (1′) and (2′). Finally, note that a continuous dipole configuration is possible between edges (2) and (2′), because edge (2) has no convex segments. Thus the whole domain structure is made up of the configuration between edges (2′) and (1′) and the fragments between edges (1) and (1′).

It is obvious that infinitely many of these configurations

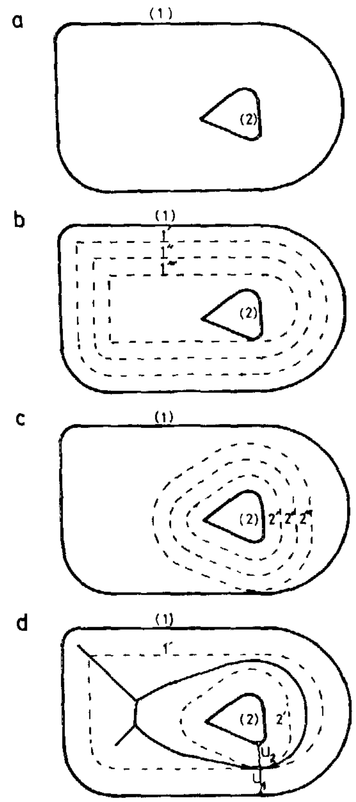

FIG. 6. (a) The object with inner edge (2) and outer edge (1). (b), (c) A few members of the family of auxiliary edges corresponding to edge (1) and edge (2), respectively. (d) The basic domain structure.

are possible for each multiply connected object, so that the basic configuration is not yet uniquely specified. For this, we hark back to a general property of basic configurations in simply connected objects, derived in the previous section. By definition, the basic domain configuration of a multiply connected object is the one in which each point at the part of the domain structure separating edges (1) and (2) is at equal distance from both edges. The basic structure can be found as follows. Determine the location where the distance between edges (1) and (2) is the smallest [the distance U_1U_2 in Fig. 6(d)]. Note that the straight line between U_1U_2 coincides with characteristics through U_1 and U_2. Construct the auxiliary edges corresponding to edges (1) and (2) passing through the middle of the line U_1U_2 and determine the domain structure as discussed above. It is obvious that, again, the basic configuration is the locus of centers of circles that touch the object edge at at least two points and that do not intersect this edge at any point.

At first sight, confinement to the basic domain structures implies overlooking a large number of alternative realizations with equal validity. However, in a forthcoming paper on composite domain structures, we shall discover that none of these possibilities is left out of consideration. Moreover, the above definition of basic domain structures in multiply connected objects allows the development of a systematic methodology to handle the composite domain structures.

Let us proceed to a more complex object with three holes (see Fig. 7), in order to develop the procedure for arbitrary multiply connected objects. In the first step, we determine the shortest distance between two of the four edges [U_3U_4 in Fig. 7(c)]. Subsequently, we construct aux-

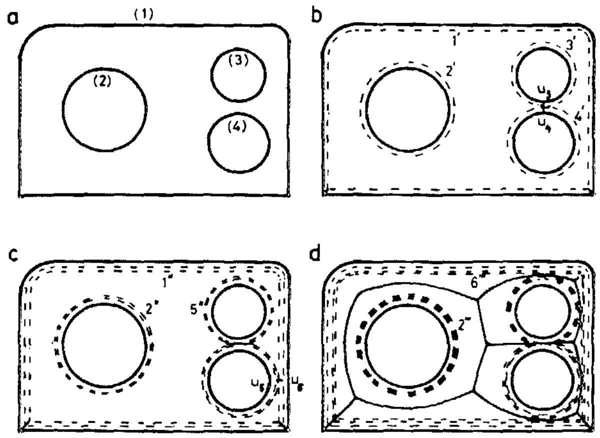

FIG. 7. The various phases in the construction of the basic structure in the object in (a).

iliary edges (1′), (2′), (3′), and (4′) at a distance $\frac{1}{2}U_3U_4$ from edges (1), (2), (3), and (4), respectively. In the next step, we construct the fragments of the domain structure between edges (1) and (1′). These fragments have their extremities in the centers of curvature of the convex parts of edge (1) with a local minimum in the radius. These wall fragments are the loci of the centers of all circles in between edges (1) and (1′) that touch edge (1) at the least two points. In the same way, the wall fragments between edges (2) and (2′), edges (3) and (3′) and edges (4) and (4′) have to be determined. Note that edges (3′) and (4′) constitute one single closed curve which we shall denote by (5′) in Fig. 7(c).

In Fig. 7(c), the shortest distance is determined between (2′), (1′), and (5′) [U_5U_6 in Fig. 7(c)]. Next, we determine the auxiliary edges (5″), (2″), and (1″) at a distance $\frac{1}{2}U_5U_6$ from edges (5′), (2′), and (1′), respectively. Again, we determine the loci of centers of circles between edges (5′) and (5″) that touch at least two points at edge (5′). The same procedure is repeated for the regions between (1′) and (1″) and between (2′) and (2″). This time, edges (1″) and (5″) constitute a closed curve, which we shall denote by (6″) in Fig. 7(d). It is self-evident that this procedure can by systematically repeated, so that we ultimately arrive at the domain structure of Fig. 7(d).

Finally, it should be mentioned that the circulation senses of **M** along the inner edges are always opposite to the circulation sense along the outermost edge.

IV. DISCUSSION

The starting point of our theory is an idealization of the material properties in which, on the micromagnetic level, the intrinsic anisotropy and the exchange energy[7] have been neglected. The latter assumption is only justified when the spatial rate in the variation of the dipole direction is sufficiently low. In general, this condition is satisfied in the domains when the lateral dimensions of the object exceed a critical value, below which the object tends to behave like a single domain particle. Even when the object is large enough, satisfaction of this requirement remains questionable inside the domain walls. However, the theory unfolded is, in the first place, based on the requirements to the micromagnetic equilibrium of the dipoles inside the domains and, as an inevitable consequence of this, domain walls, discontinuities in **M**, have to arise. No opinion is given upon the torque equilibrium within the walls; however, this subject was extensively elucidated by Hubert.[8,9] He showed that, even in these regions, magnetostatics dominates in thin-film objects.

The second requirement, the small anisotropy, is a matter of material choice and is satisfied when the ratio of the anisotropy energy constant and $\mu_0 M_s^2$ is small. A large number of materials, such as Ni, Fe, and their composites, are good representatives of this category.

A well-known basic domain structure[5,6,10–12] is the Landau–Lifshitz structure, shown in Fig. 8(a) in a Permalloy bar with a thickness of 2500 Å. It is well known that, apart from this simple configuration, a large number of alternative structures can come into being in the same object[10,11] [see Fig. 8(b)]. These type of configurations will come up for discussion in the forthcoming paper on composite domain structures. Note that the singularities in the evolute of the rectangular bar coincide with the corners. Figure 8(c) gives the basic domain structure in a bar with circular tips, in which the extremities of the central wall coincide with the centers of both circular tips. In Fig. 8(d), a simple composite structure is shown in the same object. Additional examples of basic domain structures in convex objects are provided by Fig. 9. Figure 9(a) shows a degenerated basic configuration, in which the domains structure has shrunk to one single spot in the center of the circle.

Let us proceed to objects with concave segments. Figure 10(a) shows an object bounded by two circular segments, in which the circular magnetization distributions imposed by both segments are separated by an elliptical wall, as proved previously.[13] Another example is provided by Fig. 10(b), in which the Permalloy element contains one corner covering an arc $\eta = 270°$ within the magnetic medium. Note that no domain wall arises at this corner, while both domain walls originating in the adjacent corners exhibit a parabolic course inside the sector with angle $\eta - 180°$ and center in the corner with angle η. Again, Fig. 10(b) demonstrates that the basic domain structures arise in a natural way in simply connected

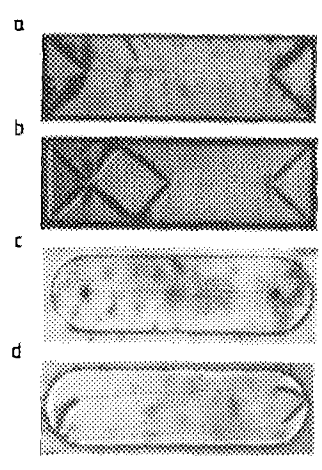

FIG. 8. Simple domain structures in Permalloy bars with a thickness of 2500 Å and a length of 50 μm. (a), (c) Basic configurations, (b), (d) simple composite structures.

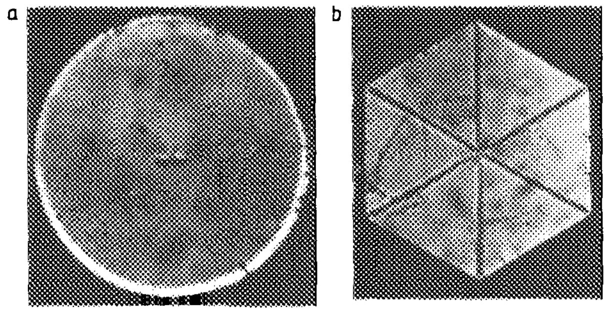

FIG. 9. Basic domain configurations in a number of convex objects (diameter circle 50 μm).

objects. Let us now have a look at multiply connected objects.

In Fig. 11(a) domain structure deviating from the basic configuration, depicted in Fig. 11(b), is shown in a circular specimen with two holes inside. The reason for this discrepancy is obviously the domain-wall energy because the actual domain structure has a smaller wall length. Moreover, observe that the actual structure is connected to the object edge, while the basic structure is a more or less floating configuration. The structure of Fig. 11(a) can easily be derived, because, in Sec. III, we have established that whole family of domain structures is possible in multiply connected objects. In order to select the one with the shortest wall length, we proceed as follows [see Fig. 11(c)].

Determine the shortest distance between the outermost edge (1) and one of the inner edges [edge (2) Fig. 11(c)]. Construct the auxiliary edge (1′) "parallel" to edge (1) that touches at edge (2). Consequently, edges (1′) and (2) constitute one closed curve which we shall label (2′). Subsequently, the shortest distance between edges (2′) and (3) is determined and, again, auxiliary edge (2″) "parallel" to (2′) that touches edge (3) is constructed. Next, the fragments of the domain walls in between edges (2′) and (2″), being the locus of centers in that region of all circles touching edge (2′) at at least two points, is determined. Finally, the basic domain structure of the region bounded by edges (2″) and (3) is added, ultimating in the configuration depicted in the photographs, although two extra edge doublets reveal

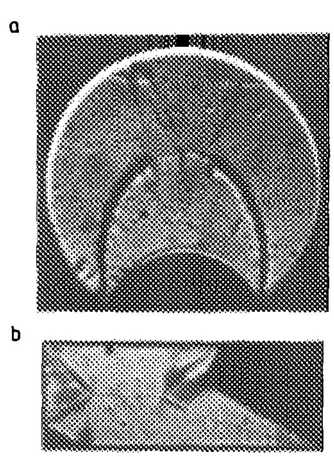

FIG. 10. Two Permalloy elements with concave segments, namely, (a) segment of a circle (diameter outer circle 50 μm) and (b) a corner with an angle inside the medium larger than 180°.

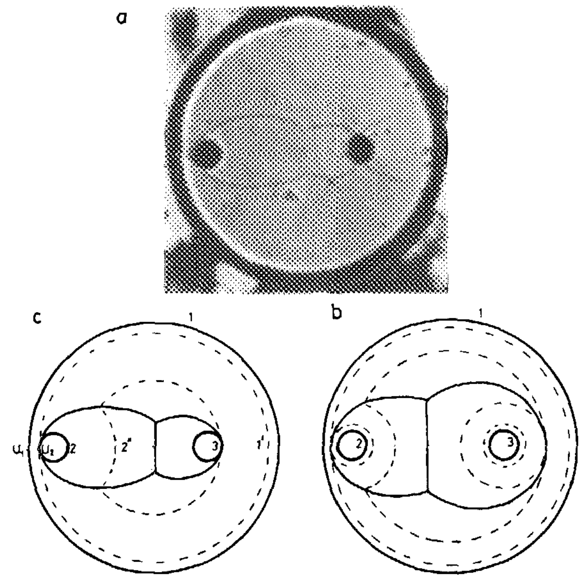

FIG. 11. (a) A multiply connected Permalloy specimen with an outer diameter of 50 μm. (b) The basic domain configuration. (c) Construction of domain structure with a configuration comparable to (b) but having a shorter wall length.

themselves at edge (3), which are symmetrically positioned with respect to the symmetry line through the centers of edges (2) and (3). One of the walls of each edge doublet belongs to the basic structure, while its counterpart extends itself into the region between edges (1) and (3). The reason for these extra walls is presumably the high-energy density, because of its high wall angle, of the domain-wall segment of the basic structure between edges (1) and (3) that touches at edges (3). This segment, having a wall angle of about 180°, is replaced by two doublet walls with greater lengths but much smaller wall angles, so that wall energy is gained. Again, the domain-wall energy is a hill joy, although it cannot affect fundamental deviations from the basic structure in this object.

The question forcing itself to the fore is whether we have made the right choice by our definition of the basic domain configurations, because it is obvious that the discrepancy observed above will always present itself in multiply connected objects. There are three reasons which justify our choice. In the first place, the description of the basic structure in terms of the locus of centers of circles within the object that touch the edge at at least two points gains general validity by this definition. In the second place, this definition of basic domain structures allows a very systematic approach to the composite domain structures, as will be shown in a forthcoming paper. Finally, it will be shown in the latter paper that the domain structure derived in the previous paragraph is just a specific example of a composite structure.

When constructing the basic configurations in multiply connected objects, we have concluded that the circulation sense of **M** along each inner edge is always opposite the sense along the outermost edge. However, there is one exception to this rule, namely, an object that consists of one inner edge running perpendicular to characteristics stretching out from

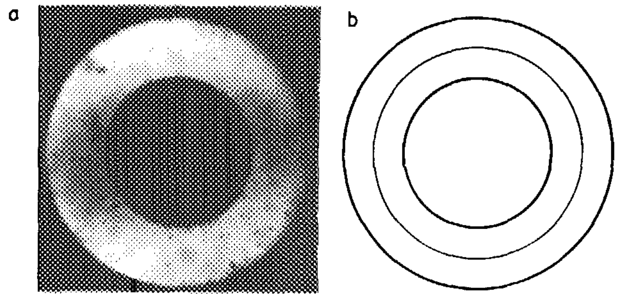

FIG. 12. The parallel configuration (a) and the basic structure (b) in a Permalloy ring with an outer diameter of 50 μm.

the outermost edge, so that no conflicting dipole distributions are imposed by inner and outer edge. An example is given in Fig. 12(a) of a Permalloy ring in which no traces of domain walls can be observed. This magnetization distribution is not a basic structure [see Fig. 12(b)] and will be called the parallel configuration. Note that we previously have met parallel configurations in Sec. III, where fragments of a wall configuration have been determined between auxiliary edges having a constant mutual distance.

A discussion of the work of Williams,[14] who confined himself to domain structures in ideally soft-ferromagnetic thin-film elements with polygonal lateral geometry, is timely here. His domain configurations exhibit a great deal of resemblance to the basic domain structures presented in this paper; however, differences reveal themselves in the case of objects with concave edge segments. In the polygonal specimens, this situation presents itself as soon as vertices are present that cover an arc larger than 180° inside the specimen. In this situation, the basic structures have shorter wall lengths and thus lower wall energy and are, therefore, more likely. This assertion is supported by Fig. (11b). It is self-evident that, because of these deviations, Williams's structures do not fit into our unifying description, in terms of loci of centers of circles. The advantage of this universal criterion will become particularly evident when the composite structures will come up for discussion in a forthcoming paper. Thus, the present approach not only covers a wider range of object geometries but, potentially, also a larger variety in the composite structures. An additional difference between the present approach and that of Williams is the uniqueness of the present construction method, while the William's involves aspects of trial and error.

ACKNOWLEDGMENTS

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APPENDIX

In this Appendix, we shall establish two properties of the circles that touch at the edge whose centers constitute the position of a domain wall of a basic domain structure. In the first place, we shall prove that both edge segments at which a particular family of circles touch constitute the mathematical envelope of this family. We have decomposed the edge into a number of segments, which are bounded by two successive points at which the radius of curvature posseses a local minimum. In the second place, we shall demonstrate that none of the circles that touch at two of these edge segments intersect these segments at any point.

Let us consider a two-parameter family of circles

$$(x-a)^2+(y-b)^2=r^2(a,b) \tag{A1}$$

and let us investigate the intersections of two circles whose centers are separated by a small distance $(\delta a, \delta b)$. When the center of the first circle is given by (a,b) the second circle satisfies

$$[x-(a+\delta a)]^2+[y-(b+\delta b)]^2 = r^2[(a+\delta a),(b+\delta b)]\,.$$

For the intersections, we obtain by substracting both equations:

$$-(x-a)\delta a-(y-b)\delta b = r\frac{\partial r}{\partial a}\delta a + r\frac{\partial r}{\partial b}\delta b + O(\delta a^2,\delta b^2)\,.$$

In a first-order approximation, we obtain

$$-\left((x-a)+r\frac{\partial r}{\partial a}\right)\delta a=\left((y-b)+r\frac{\partial r}{\partial b}\right)\delta b\,. \tag{A2}$$

Let us consider two subsequent circles that touch the edge and whose centers are separated by a distance δb, while the coordinate system is chosen such that δa is zero in a first-order approximation.

The distance of the domain wall to both edge segments is always smaller than the radii of curvature of the corresponding points because the domain wall is inside the intersection of both Ω regions. The circle whose center is in $(0, b+\delta b)$ also touches the edge and it can easily be seen that independently of the exact course of the edge segments, the first-order change in the radius (or the distance to the edge) is equal to $\cos(\psi/2)\delta b$, where ψ is the angle enclosed by the characteristics. In the limit for $\delta b\downarrow 0$, we get from Eq. (A2), for the intersections of these circles, $y-b+r\cos(\psi/2)=0$, which implies that the points of intersection coincide with the points of contact of the circle with both edge segments. In other words, the object edge is the envelope of the family of circles whose centers constitute the domain wall.

Let us pay attention to another property of these circles, which is strongly interwoven with the above observations. We distinguish between two segments on each circle, both of which are bounded by both points of contact of the circle with the edge. The smallest one, which is situated at the side of point P in Fig. 13, will be indicated by the symbol of the circle to which a prime is added, while its counterpart is indicated by a double prime. Above, we have observed that two segments, let us say $1'$ and $2'$, which are at an infinitesimally small mutual distance intersect each other in their points of contact with the edge, while the rest of these segments have, of course, no point of intersection (two circles possess at most two points of intersection). The same observation applies to the circle segments $1''$ and $2''$. Let us con-

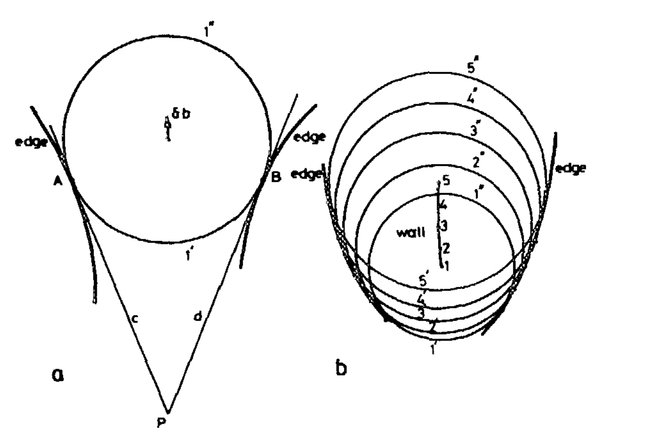

FIG. 13. (a) The partitioning into two segments 1′ and 1″ of the circle with center 1 that touches the edge at A and B. (b) The correlation between the order in the centers of the circles and the segments into which these circles have been partitioned.

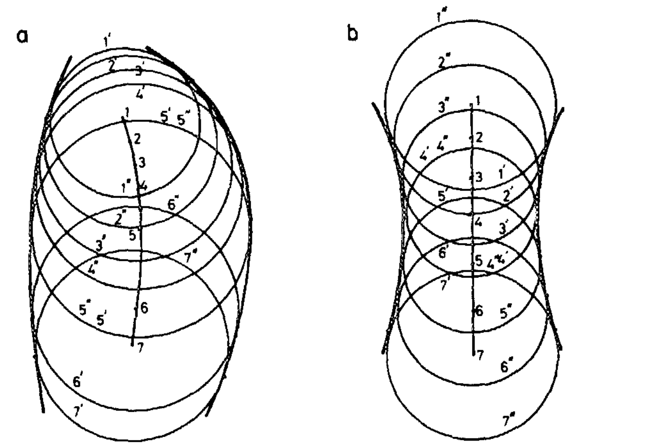

FIG. 14. The course of the two categories of circle segments on both sides of the circle whose corresponding tangents to the edge are parallel: (a) two convex edge segments and (b) two concave edge segments.

sider a set of circles that touch at both edge segments whose centers on the domain wall are numbered consecutively. It is obvious that the circle segments that are at a finite mutual distance, let us say 1′ and 3′, or 1″ and 3″, do not intersect. Moreover, it should be observed that the circle segments of both categories exhibit the same order as the corresponding centers [see Fig. 13(b)].

This order is not disturbed as long as the smaller circle segments remain on the same side of their corresponding centers. Both categories change parts as soon as the tangents to the edge are parallel and point P in Fig. 14(a) shifts towards infinity. Let's have a closer look at this situation. In all cases, the edge segments are located outside the circle that touches at both edge segments, for, its radius is always smaller than the radius of curvature of one of these convex segments. It can be seen from Fig. 14(a) that the above-distinguished categories of circle segments interchange parts at the circle marked by 5, whose corresponding tangents to the edge are parallel. It is obvious from the above established correlation between the order in the centers of the circles and both categories of circle segments that segments 6″ have no intersection with segments 1′, 2′, 3′, and 4′, and the same holds for the circle segments 5″, 6″, 7″, 3′, 2′, and 1′ in Fig. 14. Therefore, we can draw the conclusion that when both outermost circles that touch at the extremities of two edge segments are located within an object, all other circles that touch at both segments are also entirely situated within the object.

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