La situación que estudiamos es esquemáticamente ⇒

{Espectro de autovalores de H_0} {Autovalor de $H = H_0 + \underline{\lambda V}$} {Autovalores próximos a E}

$$(E - E_n^{(0)})\, c_n = \sum_m V_{nm}\, c_m \qquad (1)$$

(A) Aproximaciones simples: (Naive perturbation theory)

El autovalor $E = E_k$ es suficientemente próximo a $E_k^{(0)}$ (autovalor de H_0)

$$\Rightarrow E_k = E_k^{(0)} + \lambda E_k^{(1)} + \lambda^2 E_k^{(2)} + \dots$$

y la función de onda tiene coeficientes

$$c_k = 1 + \lambda c_k^{(1)} + \lambda^2 c_k^{(2)} + \dots$$

reemplazando en la ecuación (1) tenemos

$$(\cancel{E_k^{(0)}} + \lambda E_k^{(1)} + \lambda^2 E_k^{(2)} + \dots - \cancel{E_k^{(0)}})(1 + \lambda c_k^{(1)} + \lambda^2 c_k^{(2)} + \dots)$$

$$= \lambda \sum_m V_{km} (1 + \lambda c_m^{(1)} + \lambda^2 c_m^{(2)} + \dots)$$

$$\Rightarrow \lambda E_k^{(1)} = \lambda V_{kk} \rightarrow E_k^{(1)} = V_{kk}$$

$$\lambda^2 E_k^{(2)} + \lambda^2 E_k^{(1)} c_k^{(1)} = \lambda^2 \sum_m V_{km} c_m^{(1)}$$

$$\Rightarrow E_k^{(2)} = \sum_{m \neq k} V_{km} c_m^{(1)}$$

$$\Rightarrow \quad E = H_{11} - \sum_{k=2} \frac{|H_{1k}|^2}{(H_{1k} - E)}$$

Primera aproximación $\Rightarrow$ $E = H_{11} = E_1^{(0)} + V_{11}$

$$E = E_1^{(0)} + V_{11} + \sum_{k=2} \frac{|V_{1k}|^2}{(E_1^{(0)} + V_{11} - (E_k^{(0)} + V_{kk}))}$$

$$E = E_1^{(0)} + V_{11} + \sum_{k=2} \frac{|V_{1k}|^2}{\left[(E_1^{(0)} - E_k^{(0)}) + (V_{11} - V_{kk})\right]}$$

$$\Rightarrow \quad E \simeq E_1^{(0)} + V_{11} + \sum_{k=2} \frac{|V_{1k}|^2}{(E_1^{(0)} - E_k^{(0)})}$$

HASTA ahora hemos considerado niveles de energía suficientemente separados.
Veamos qué ocurre cuando los niveles están próximos (pero no degenerados).
Tomamos dos niveles

$$E_2 = E_2^{(0)}$$
$$E_1 = E_1^{(0)}$$

$$H_0 \varphi_1 = E_1 \varphi_1$$
$$H_0 \varphi_2 = E_2 \varphi_2$$

Niveles no perturbados

$$\psi = a\varphi_1 + b\varphi_2$$
$$H = H_0 + V$$
$$(H_{11} - E)a + H_{12} b = 0$$
$$(H_{21})a + (H_{22} - E) b = 0$$

$$\Rightarrow (H_{11} - E)(H_{22} - E) - |H_{12}|^2 = 0$$

⇒ Primer orden

$$\begin{cases} E_n^{(1)} = V_{nn} = \int \varphi_n^{(0)*} V \varphi_n^{(0)} d\tau \\ \varphi_n = \varphi_n^{(0)} + \sum_{k \neq n} \left(\frac{V_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) \varphi_k^{(0)} \end{cases}$$

⇒ Segundo orden

$$E_k^{(2)} = \sum_{m \neq k} \frac{V_{km} V_{mk}}{E_k^{(0)} - E_m^{(0)}} = \sum_{m \neq k} \frac{|V_{km}|^2}{(E_k^{(0)} - E_m^{(0)})}$$

$$\Rightarrow \left[E_k = E_k^{(0)} + V_{kk} + \sum_{m \neq k} \frac{|V_{km}|^2}{(E_k^{(0)} - E_m^{(0)})} \right.$$

Condiciones de aplicabilidad de la teoría de perturbaciones ⇒

a) $|V_{km}| \ll |E_k^{(0)} - E_m^{(0)}|$

b) $\delta E^{(2)} = \sum_{m \neq k} \frac{|V_{km}|^2}{(E_k^{(0)} - E_m^{(0)})} < 0$

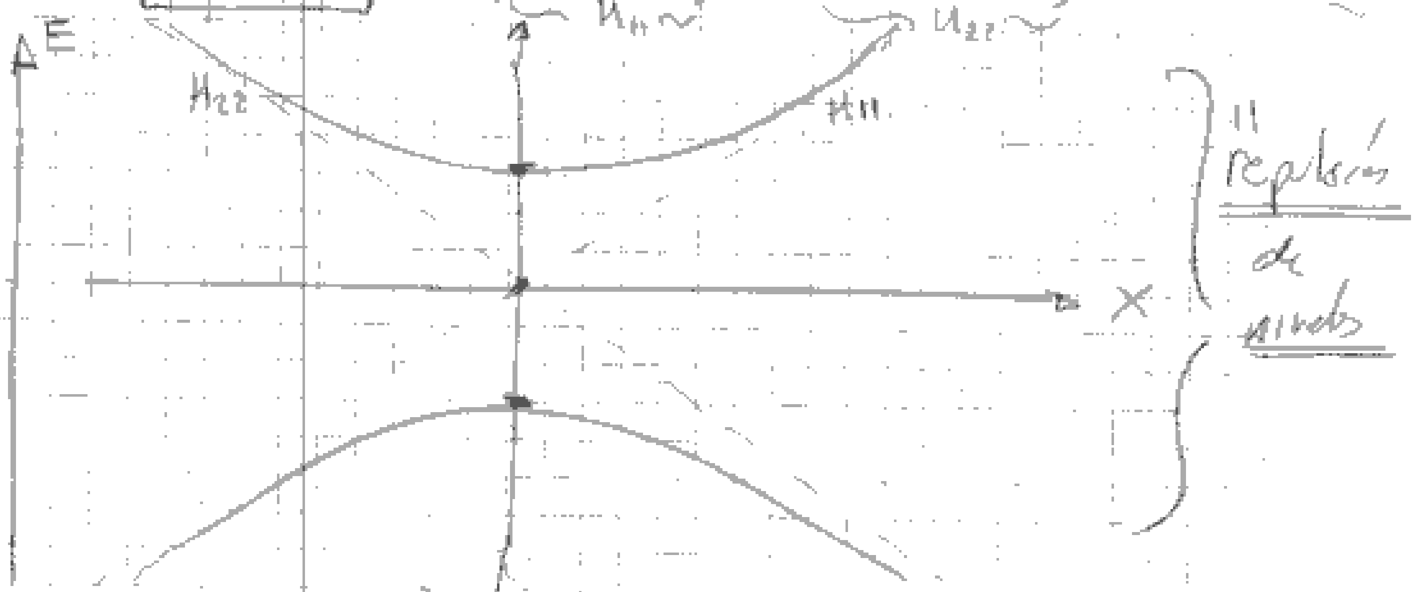
Veamos, como tratamiento alternativo, el "desarrollo matricial".

Si, en cambio, $|H_{11}-H_{22}| \ll |V_{12}|$

$$\Rightarrow \begin{cases} E_\pm = \frac{1}{2}(H_{11}+H_{22}) \pm |V_{12}|\left(1 + \frac{(H_{11}-H_{22})^2}{4|V_{12}|^2}\right)^{1/2} \\ E_\pm = \frac{1}{2}(H_{11}+H_{22}) \pm |V_{12}| \pm \frac{(H_{11}-H_{22})^2}{8|V_{12}|^2} \end{cases}$$

Veamos el comportamiento de estos valores ⇒

$$X = \underbrace{H_{11}-H_{22}}_{} = (\underbrace{E_1^{(0)}+V_{11}}_{H_{11}}) - (\underbrace{E_2^{(0)}+V_{22}}_{H_{22}})$$



Con $E_1^{(0)}$ y $E_2^{(0)}$ y a V_{11} y V_{22}

Teoría de perturbaciones para niveles degenerados ⇒

Si un dado nivel $E_\alpha^{(0)}$ está Ω-veces degenerado la ecuación característica es de la forma

$$\sum_{k=1}^{\Omega}(H_{mk}-E_\alpha\,\delta_{mk})\,a_k = 0 \qquad m = 1 \to \Omega$$

y procedemos exactamente como en el caso anterior.

$$\det \begin{vmatrix} (H_{11}-E) & H_{12} & H_{13} & \cdots & H_{1N} \\ H_{21} & (H_{22}-E) & 0 & \cdots & 0 \\ H_{31} & 0 & (H_{33}-E) & \cdots & 0 \\ \vdots & & & & \\ H_{N1} & 0 & 0 & \cdots & (H_{NN}-E) \end{vmatrix} = 0$$

Tomemos el caso de 3 componentes

$$\begin{vmatrix} (H_{11}-E) & H_{12} & H_{13} \\ H_{21} & (H_{22}-E) & 0 \\ H_{31} & 0 & (H_{33}-E) \end{vmatrix}$$

$$= (H_{11}-E)(H_{22}-E)(H_{33}-E) - H_{12}H_{21}(H_{33}-E) - H_{13}H_{31}(H_{22}-E)$$

$$\Rightarrow (H_{11}-E) - \frac{H_{12}H_{21}}{(H_{22}-E)} - \frac{H_{13}H_{31}}{(H_{33}-E)} = 0$$

$$\Rightarrow H_{11} - E = \sum_{k=2} \frac{H_{1k}H_{k1}}{(H_{kk}-E)}$$

$$E = H_{11} - \sum_{k=2} \frac{|H_{1k}|^2}{(H_{kk}-E)}$$

Esta ecuación puede ser resuelta por medio de aproximaciones sucesivas $\Rightarrow$

Efecto Zeeman

La perturbación que experimenta un átomo en presencia de un campo externo se escribe

$$V = \left(\frac{-e}{2mc}\right)\left\{ \mathcal{H}\cdot(\bar{L}+2\bar{S})\right\}$$

Para un dado nivel (J, E_J) tenemos

$$\langle E_J J M | V | E_J J M' \rangle$$

Como $\bar{J} = \bar{L} + \bar{S}$

el operador $\bar{L} + 2\bar{S} = (\bar{L}+\bar{S}) + \bar{S} = (\bar{J} + \bar{S})$

es "proporcional" a $\bar{J}$ en el subespacio de estados con buen $\bar{J}$

$$\Rightarrow (\bar{L}+2\bar{S}) \longrightarrow g(\bar{J})$$

de donde $\bar{\mathcal{H}}\cdot(\bar{L}+2\bar{S}) = g\,\mathcal{H}_z J_z$

$$\langle E_J, J, M | V | E_J, J, M' \rangle$$

$$= g\left(-e/2mc\right)\mathcal{H}\,\langle E_J, J, M | J_z | E_J, J M' \rangle$$

$$= -\left(\frac{e\mathcal{H}}{2mc}\right) g\, M\, \delta_{M,M'}$$

$$\Delta E(\text{primer orden}) = -\mathcal{H} M g \mu_{Bohr}\, \delta_{M,M'}$$

$$E(J,M) = E_J - \mathcal{H} g \mu_{Bohr} M$$

Determinación de g (Factor de Landé)

$$\Rightarrow \quad H_{11}H_{22} + E^2 - E(H_{11} + H_{22}) - |H_{12}|^2 = 0$$

$$E = \frac{1}{2}(H_{11} + H_{22}) \pm \frac{1}{2}\left[(H_{11} + H_{22})^2 + 4|H_{12}|^2 - 4H_{11}H_{22}\right]^{1/2}$$

$$E = \frac{1}{2}(H_{11} + H_{22}) \pm \frac{1}{2}\left[(H_{11} - H_{22})^2 + 4|H_{12}|^2\right]^{1/2}$$

Se lo trata para $|H_{11} - H_{22}| \gg |H_{12}|$

$$\Rightarrow \quad E = \frac{1}{2}(H_{11} + H_{22}) \pm \frac{1}{2}(H_{11} - H_{22})\left(1 + \frac{4|H_{12}|^2}{(H_{11} - H_{22})^2}\right)^{1/2}$$

$$E_+ \cong \frac{1}{2}(H_{11} + H_{22}) + \frac{1}{2}(H_{11} - H_{22})\left(1 + \frac{2|H_{12}|^2}{(H_{11} - H_{22})^2}\right)$$

$$E_- \cong \frac{1}{2}(H_{11} + H_{22}) - \frac{1}{2}(H_{11} - H_{22})\left(1 + \frac{2|H_{12}|^2}{(H_{11} - H_{22})^2}\right)$$

$$\begin{cases} E_+ = H_{11} + \dfrac{|H_{12}|^2}{(H_{11} - H_{22})} \\[2ex] E_- = H_{22} - \dfrac{|H_{12}|^2}{(H_{11} - H_{22})} \end{cases}$$

$$H_{11} - H_{22} = E_1^{(0)} - E_2^{(0)} + (V_{11} - V_{22}) = (E_1^{(0)} + V_{11}) - (E_2^{(0)} + V_{22})$$

$$H_{12} = V_{12}$$

$$E_+ = E_1^{(0)} + V_{11} + \frac{|V_{12}|^2}{(E_1^{(0)} + V_{22}) - (E_2^{(0)} + V_{22})}$$

$$E_- = E_2^{(0)} + V_{22} + \frac{|V_{12}|^2}{(E_2^{(0)} + V_{22}) - (E_1^{(0)} + V_{11})}$$

$$\begin{pmatrix} l' & 1 & l \\ -m' & 0 & m \end{pmatrix} \neq 0 \quad \text{solo si} \quad \begin{cases} l' = l \pm 1 \\ m_l' = m \end{cases}$$

(a) $l' = l+1$ $\qquad (-)^{l+1+l'} = +1$

$$\begin{pmatrix} l+1 & 1 & l \\ m & 0 & -m \end{pmatrix} = \begin{pmatrix} l & 1 & l+1 \\ -m & 0 & m \end{pmatrix} = \begin{pmatrix} l & 1 & l+1 \\ m & 0 & -m \end{pmatrix}$$

$$\Rightarrow \begin{cases} j = l, \; m = m \\ s = 1, \; \mu = 0 \\ e = 1 \\ \cancel{\phi}^{1}_{01} \end{cases} \Rightarrow (-)^{l+m} \left\{ \frac{(2l)!\,(l+m+1)!}{(2l+3)!\,(l+m)!} \right\}^{1/2}$$

$$(-)^{1} \left\{ 2(l-m+1) \right\}^{1/2}$$

$$= (-)^{1+l+m} \left[\frac{2(l+m+1)(l-m+1)}{(2l+3)(2l+2)(2l+1)} \right]^{1/2}$$

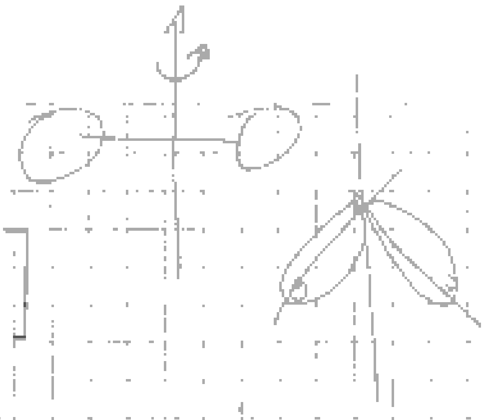
$$\begin{pmatrix} l+1 & 1 & l \\ 0 & 0 & 0 \end{pmatrix} = \cancel{(-)^{1+l}} \left[\frac{2(l+1)(l-1)}{(2l+3)(2l+2)(2l+1)} \right]^{1/2}$$

$$\Rightarrow \begin{pmatrix} l+1 & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l+1 & 1 & l \\ m & 0 & -m \end{pmatrix}$$

$$= \cancel{2} \left[(l^2-1)\left[(l+1)^2 - m^2\right] \right]^{1/2} \cdot \frac{(-)^m}{\cancel{2}(l+1)(2l+1)(2l+3)} \cdot$$

$$M_{l'=l+1,\,m,\,l,\,m} = -P_c \frac{\varepsilon \left[(2l+1)(2l+3)\right]^{1/2} (l+1)^{1/2} (l-1)^{1/2}}{(l+1)(2l+1)(2l+3)} \cdot \left[(l+1)^2 - m^2\right]^{1/2}$$

Idem $M_{l'=l-1,\,m,\,l,\,m}$



Aplicaciones

[– Efecto Stark
– Efecto Zeeman
– Efecto Paschen – Back]

▷ Efecto STARK →

Consideremos un rotor rígido ⇒ (Molécula diatómica, p ejemplo)

$$H_0 = \frac{\hat{L}^2}{2I}$$

$$\varphi_{\ell m_\ell} = Y_{\ell m_\ell}(\theta,\varphi) \qquad \int d\Omega\, Y^*_{\ell' m_\ell'} Y_{\ell m_\ell} = \delta_{\ell\ell'}\delta_{m_\ell m_\ell'}$$

$$E_0(\ell) = \frac{\hbar^2}{2I}\ell(\ell+1)$$

Perturbamos: $V = -\vec{p}_e\cdot\vec{E} = -p_e E\cos\theta$

$$M_{\ell' m_\ell', \ell m_\ell} = \langle \ell' m_\ell' | V | \ell m_\ell \rangle = \int d\Omega\, Y^*_{\ell' m_\ell'}(\Omega)\,(-p_e E\cos\theta)\, Y_{\ell m_\ell}(\Omega)$$

$$= -p_e E \int d\Omega\, Y^*_{\ell' m_\ell'}(\theta,\varphi)\cos\theta\, Y_{\ell m_\ell}(\theta,\varphi)$$

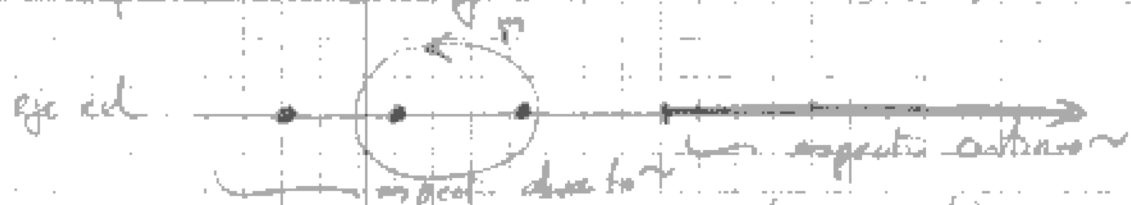
$$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta\, Y^*_{\ell' m_\ell'}(\theta,\varphi)\, Y_{10}(\theta,\varphi)\, Y_{\ell m_\ell}(\theta,\varphi)$$

$$M_{\ell' m_\ell', \ell m_\ell} = \left(-p_e E\sqrt{\frac{4\pi}{3}}\right)\left[\frac{(2\ell+1)(2\ell'+1)(3)}{4\pi}\right]^{1/2} \begin{pmatrix} \ell' & 1 & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell' & 1 & \ell \\ -m_\ell' & 0 & m_\ell \end{pmatrix}(-)^{m_\ell'}$$

▷ Forma implícita del desarrollo perturbativo a todos órdenes

Representamos el conjunto de autovalores de H:



$$G(z) = \frac{1}{z - H} \qquad \text{(resolvente)}$$

$E_i \Rightarrow$ valores propios

$\hat{P}_i \Rightarrow$ {proyector sobre el subespacio correspondiente al valor propio E_i}

$$\sum_i \hat{P}_i = \hat{I}$$

$$H\Psi = E\Psi$$

Definición

$$H(P_i\Psi) = E_i P_i \Psi$$

$$HP_i = E_i P_i$$

$$G(z)P_i = \frac{P_i}{(z - E_i)} \quad \Rightarrow \quad G(z) = \sum_i \frac{P_i}{(z - E_i)}$$

Para cada autovalor discreto E_i de H, este es un polo simple de $G(z)$ y el residuo es P_i

$$P_i = \frac{1}{2\pi i} \oint_{\Gamma_i} G(z)\, dz$$

en un contorno cerrado de E_i

$\Rightarrow$ para todos los contornos

$$\Rightarrow \left(\frac{M^2_{\ell' m, \ell m}}{E_\ell - E_{\ell'}} \right)$$

	$E_{\ell'}$	E_ℓ	$D_{\ell\ell'}$
$\ell' = \ell-1 \rightarrow$	$((\ell-1)\ell)$	$\ell(\ell+1)$	$\ell(\ell+1) - \ell(\ell-1)$
$\ell' = \ell+1 \rightarrow$	$(\ell+1)(\ell+2)$	$\ell(\ell+1)$	$\ell(\ell+1) - (\ell+1)(\ell+2)$

$$\Delta \mathcal{E} \rightarrow \frac{2\mathcal{I}}{\hbar^2}(p_e \mathcal{E})^2 \left[\frac{\langle \ell m | \cos\theta | \ell+1, m\rangle^2}{\ell(\ell+1) - (\ell+1)(\ell+2)} + \frac{\langle \ell m | \cos\theta | \ell-1, m\rangle^2}{\ell(\ell+1) - \ell(\ell-1)} \right]$$

$$\langle \ell m | \cos\theta | \ell+1, m\rangle = \langle \ell-1, m | \cos\theta | \ell, m\rangle$$

$$= (\ell^2 - m^2)^{1/2} \cdot \frac{1}{(4\ell^2 - 1)^{1/2}} \quad \Leftarrow [\text{Ejercicio}]$$

$$\Delta E^{(2)} \rightarrow \frac{2\mathcal{I}}{\hbar^2}(p_e \mathcal{E})^2 \frac{(\ell^2 - m^2)}{(4\ell^2 - 1)} \left[\frac{1}{\ell(\ell+1) - (\ell+1)(\ell+2)} + \frac{1}{\ell(\ell+1) - \ell(\ell-1)} \right]$$

$\Rightarrow$ {Corrección a segundo orden dependiente de $\underline{\underline{m^2}}$}

Por lo tanto

$$\begin{cases} \rho|\tilde\alpha\rangle_0 = \sum_\beta |\beta\rangle(\langle\beta|\tilde\alpha\rangle_0) = \sum_\beta |\beta\rangle\,({}_0\langle\beta|\tilde\alpha\rangle_0) \\ \qquad\quad = \sum_\beta |\beta\rangle\,\delta_{\alpha\beta} = |\alpha\rangle \\ \rho_0\,|\alpha\rangle = \sum_\beta |\beta\rangle_0\,{}_0\langle\tilde\beta|\alpha\rangle = \sum_\beta |\beta\rangle_0\,\delta_{\alpha\beta} = |\alpha\rangle_0 \end{cases}$$

Sea $U|\alpha\rangle_0 = |\alpha\rangle$

$\mathcal{P}\,|\tilde\alpha\rangle_0 = |\alpha\rangle$

En la [illegible] $U|\,\rangle = \mathcal{P}|\,\rangle = 0$
si $|\,\rangle$ es un vector ortogonal a cualquier vector de Ω_0

Trivialmente

$$\begin{cases} U = \sum_\alpha |\alpha\rangle\,{}_0\langle\alpha| = \sum_\alpha |\alpha\rangle\,{}_0\langle\tilde\alpha| \\ \mathcal{P} = \sum_\alpha |\alpha\rangle\,{}_0\langle\tilde\alpha| \end{cases}$$

Obs: $\mathcal{P}\rho_0 = \sum_{\beta\gamma} |\beta\rangle(\langle\tilde\beta|\tilde\gamma\rangle_0)\,{}_0\langle\gamma| = \sum_\beta |\beta\rangle\,{}_0\langle\beta|$

Autofunciones $\Rightarrow$

Sea $A = \rho_0 V U$

(Vector en Ω_0)

$\rho_0 V U = \rho_0 (H - H_0) U$

$$\vec{L} = \sum_i \vec{l}_i \qquad \vec{S} = \sum_i \vec{s}_i$$

$$\vec{J}\cdot(\vec{L}+2\vec{S}) = (\vec{L}+\vec{S})\cdot(\vec{L}+2\vec{S}) = L^2 + 3\vec{L}\vec{S} + 2S^2$$

$$J^2 = L^2 + S^2 + 2\vec{L}\vec{S}$$

$$\vec{J}\cdot(\vec{L}+2\vec{S}) = (L^2+S^2+2\vec{L}\cdot\vec{S}) + S^2 + \vec{L}\cdot\vec{S}$$

$$= J^2 + S^2 + \tfrac{1}{2}(J^2 - L^2 - S^2)$$

$$\Rightarrow \langle \vec{J}\cdot(\vec{L}+2\vec{S})\rangle = \hbar^2\Big[J(J+1) + S(S+1) + \tfrac{1}{2}J(J+1) - \tfrac{1}{2}L(L+1) - \tfrac{1}{2}S(S+1)\Big]$$

$$= \hbar^2\Big[\tfrac{3}{2}J(J+1) + \tfrac{1}{2}S(S+1) - \tfrac{1}{2}L(L+1)\Big]$$

$$\Rightarrow \langle \vec{J}^2 \rangle = J(J+1)\hbar^2$$

$$\Rightarrow g\,J(J+1) = J(J+1) + \tfrac{1}{2}\big[J(J+1) + S(S+1) - L(L+1)\big]$$

$$g = 1 + \frac{1}{2J(J+1)}\{J(J+1) + S(S+1) - L(L+1)\}.$$

(Efecto Paschen – Back) → Ruptura de la degeneración $(2S+1)(2L+1)$

$$\langle \alpha L S m_L m_S | V | \alpha L S m_L' m_S' \rangle = -\mathcal{H}\mu_B (m_L + 2m_S)\,\delta_{m_L m_L'}\delta_{m_S m_S'}$$

$$P = \frac{1}{2\pi i}\oint G_0(z)\,dz + \sum_{n=1}^{\infty} \lambda^n P^{(n)}$$

$$P^{(n)} = \frac{1}{2\pi i}\oint G_0 (V G)^n \, dz$$

$$P = P^{(0)} + \sum_{n=1}^{\infty} \lambda^n P^{(n)}$$

Método de Kato:

Utilizando los polos de G_0 $(E_a^{(0)})$

$$G_0 = \underbrace{\frac{P^{(0)}}{z - E_a^{(0)}}}_{\text{Polo simple}} + \underbrace{\sum_{k=1}^{\infty} (-)^{k-1} (z - E_a^{(0)})^{k-1} \cdot \frac{Q^{(0)}}{a^k}}_{\text{multiplicidad}}$$

$$G_0 = \sum_{k=0}^{\infty} (-)^{k-1} (z - E_a^{(0)})^{k-1} S^{(k)}$$

$$S^{(k)} = \begin{cases} -P^{(0)} & k=0 \\ Q^{(0)}/a^k & k \geq 1 \end{cases}$$

$$\Rightarrow P = P^{(0)} + \lambda\left(P^{(0)} V \frac{Q^{(0)}}{a} + \frac{Q^{(0)}}{a} V P_0\right)$$

$$+ \lambda^2 \Big(P_0 V \frac{Q^{(0)}}{a} V \frac{Q^{(0)}}{a} + \frac{Q^{(0)}}{a} V P^{(0)} V \frac{Q^{(0)}}{a}$$

$$+ \frac{Q^{(0)}}{a} V \frac{Q^{(0)}}{a} V P^{(0)}$$

$$- P^{(0)} V P^{(0)} V \frac{Q^{(0)}}{a^2} - P^{(0)} V \frac{Q^{(0)}}{a^2} V P^{(0)}$$

$$- \frac{Q^{(0)}}{a^2} V P^{(0)} V P^{(0)} \Big) + \cdots$$

$$P_I = \frac{1}{2\pi i}\oint_I G(z)\,dz$$

$$(z-H)G = G(z-H) = 1 \quad \Rightarrow \quad zG = 1 + HG$$

$$HP_I = \frac{1}{2\pi i}\oint_I z\,G(z)\,dz \qquad \left(\oint 1\,dz = 0\right)$$

Teoría de Perturbaciones $\Rightarrow$

$$G = \frac{1}{(z - H_0 - \lambda V)}$$

$$G_0 = \frac{1}{(z-H_0)}$$

$$\frac{1}{z-H_0-\lambda V} = \frac{1}{(z-H_0)}\left[(z-H_0-\lambda V)+(\lambda V)\right]\left(\frac{1}{z-H_0-\lambda V}\right)$$

$$= \frac{1}{(z-H_0)} + \frac{1}{(z-H_0)}\lambda V \frac{1}{(z-H_0-\lambda V)}$$

$$G = G_0 + G_0 \lambda V G$$

$$G = G_0(1+\lambda V G)$$

$\Rightarrow$ A todo orden en teoría de perturbaciones

$$G = \sum_{n=0}^{\infty} \lambda^n G_0 (V G_0)^n$$

ya que iterando $\Rightarrow G = G_0(1+\lambda V G_0(1+\lambda V G_0 \cdots))$

sustituyendo en la expresión del proyector

Si $B|\bar{\alpha}\rangle_0 = |\alpha\rangle_0$

$\Rightarrow B^{-1}|\alpha\rangle_0 = |\bar{\alpha}\rangle_0$ (invertible)

$\Rightarrow \mathcal{A} = AB^{-1}$

deja de ser hermítico en Ω_0

(no posee funciones propias ortogonales)

Ejemplo simple $\Rightarrow$

Sea la ecuación de Mathieu

$$\begin{cases} \left(-\dfrac{d^2}{d\theta^2} + s\cos^2\theta\right)\psi(\theta) = E\,\psi(\theta) \\ \psi(\theta+2\pi) = \psi(\theta) \end{cases}$$

$$\left[\begin{array}{l} H_0 = -\dfrac{d^2}{d\theta^2} \\ V = s\cos^2\theta \\ E_0^{(n)} = n^2 \end{array}\right.$$

$$\begin{cases} \varphi_0^{(+)} = 1/\sqrt{\pi}\,, & \varphi_n^{(+)} = \dfrac{1}{\sqrt{\pi}}\cos n\theta \\ & ,\ \varphi_n^{(-)} = \dfrac{1}{\sqrt{\pi}}\sin n\theta \end{cases}$$

Definición y propiedades de los operadores P y Q

Operadores de proyección $\rightarrow$ $H_0 P_0 = P_0 H_0 = E_0 P_0$

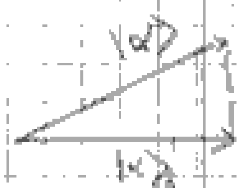
$$P = \sum_\alpha |\alpha\rangle\langle\alpha|$$

$$Q_0 = 1 - P_0$$

$(|\alpha\rangle_0 , |\tilde{\alpha}\rangle_0)$ ...

Análogamente $Q = 1 - P$

Los espacios Ω y Ω_0 no son ortogonales



$\Rightarrow$ $|\alpha\rangle_0 = P_0|\alpha\rangle$ $\left\{ \text{Proyección de } |\alpha\rangle \text{ sobre el subespacio } \Omega_0 \right\}$

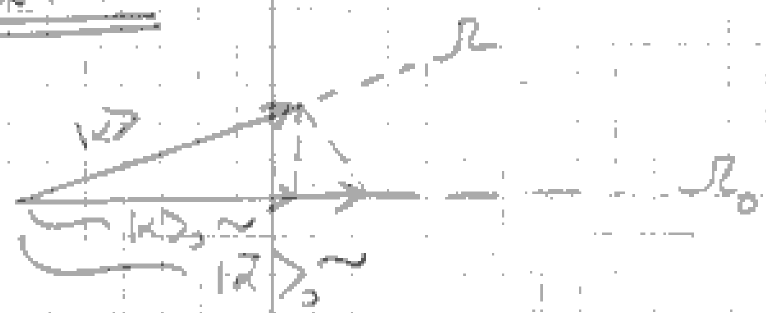
$|\alpha\rangle = P\,|\tilde{\alpha}\rangle_0$ $\left\{ \text{Proyección de } |\tilde{\alpha}\rangle_0 \text{ sobre } \Omega \right\}$

Bi-ortogonal $\Rightarrow$ $\langle\alpha|\tilde{\beta}\rangle_0 = \delta_{\alpha\beta}$

$\{ |\alpha\rangle_0 \Rightarrow$ conjunto linealmente independiente en $\Omega_0 \}$.

$\{ |\tilde{\alpha}\rangle_0 \Rightarrow$ conjunto linealmente independiente asociado a los vectores "complementarios" $\{ |\tilde{\alpha}\rangle_0 \} \}$.

Resumos



$$P = \sum_\alpha |\alpha\rangle\langle\alpha|$$

$$P_0 = \sum_\alpha |\alpha\rangle_0 \; {}_0\langle\tilde{\alpha}| = \sum_\alpha |\tilde{\alpha}\rangle_0 \; {}_0\langle\alpha|$$

$$\mathcal{U} = \sum_\alpha |\alpha\rangle \; {}_0\langle\tilde{\alpha}|$$

$$\mathcal{P} = \sum_\alpha |\alpha\rangle \; {}_0\langle\alpha|$$

$$\mathcal{A} = P_0 V \mathcal{U}$$

$$(\mathcal{A} - E_\alpha)|\alpha\rangle_0 = 0$$

$\Rightarrow \Delta E_\alpha$ = desvio a $E_\alpha^{(0)}$ devido à interacção V

$$A = P_0 V \mathcal{P}$$

$$B = P_0 \mathcal{P}$$

$$(A - B E_\alpha)|\tilde{\alpha}\rangle_0 = 0$$

$$\mathcal{A} = A B^{-1}$$

Correspondência entre métodos

$$(H - E_0)|\alpha\rangle = E_\alpha|\alpha\rangle$$

$$P_0(H - E_0)|\alpha\rangle = P_0(H_0 + V - E_0)|\alpha\rangle =$$

$$= P_0 V|\alpha\rangle$$

$$= E_\alpha|\alpha\rangle_0$$

$$= \rho_0 (H - H_0) \sum_\alpha |\alpha\rangle {}_0\langle\alpha|$$

$$= \rho_0 \sum_\alpha \left[H|\alpha\rangle - H_0 |\alpha\rangle \right] ({}_0\langle\alpha|)$$

$$\begin{cases} H|\alpha\rangle = (H_0 + V)|\alpha\rangle = (E_0 + E_\alpha)|\alpha\rangle \\ H_0 |\alpha_0\rangle = E_0 |\alpha_0\rangle \end{cases}$$

$$A = \rho_0 \left(\sum_\alpha (|\alpha\rangle) E_\alpha ({}_0\langle\alpha|) \right)$$

$$\Rightarrow (A - E_\alpha) |\alpha\rangle_0 = 0$$

$$(A - E_\alpha) |\alpha\rangle_0 = (A - E_\alpha \rho_0) |\alpha\rangle_0$$

$$= \sum_\beta |\beta\rangle_0 (E_\beta - E_\alpha) ({}_0\langle\beta|\alpha\rangle_0)$$

$$= \sum_\beta |\beta\rangle_0 (E_\beta - E_\alpha) \delta_{\alpha\beta} = 0$$

Análogamente, podemos definir a ρ_0

$$A = \sum_\alpha (|\alpha\rangle_0) E_\alpha ({}_0\langle\alpha|)$$

$$B = \sum_\alpha (|\alpha\rangle_0)({}_0\langle\alpha|)$$

$$(A - B E_\alpha)|\alpha\rangle_0 = \sum_\beta \left\{ |\beta\rangle_0 {}_0\langle\beta|\alpha\rangle_0 E_\beta - E_\alpha |\beta\rangle_0 {}_0\langle\beta|\alpha\rangle_0 \right\}$$

$$= 0$$

Las funciones propias se pueden agrupar en 4 grupos

$$\begin{cases} \psi^{(+)}_{(2n)} = \varphi_0 , \dfrac{1}{\sqrt{\pi}} \cos(2n)\theta & [n=0,1,2,\dots] \\ \psi^{+}_{(2n+1)} = \dfrac{1}{\sqrt{\pi}} \cos(2n+1)\theta \end{cases}$$

$$\begin{cases} \psi^{-}_{(2n+2)} = \dfrac{1}{\sqrt{\pi}} \operatorname{sen}(2n+2)\theta \\ \psi^{-}_{(2n+1)} = \dfrac{1}{\sqrt{\pi}} \operatorname{sen}(2n+1)\theta \end{cases}$$

$$\begin{cases} \langle n|V|n\rangle = \dfrac{S}{2} & (n=0,2,4,\dots) \\ \langle 0|V|2\rangle = \langle 2|V|0\rangle = \dfrac{S}{\sqrt{8}} \\ \langle n|V|n+2\rangle \, \langle n+2|V|n\rangle = \dfrac{S}{4} & (n=2,4,\ ) \end{cases}$$

$$A = P_0 V P_0 + P_0 V \frac{Q_0}{a} V P_0 + P_0 V \frac{Q_0}{a} V \frac{Q_0}{a} V P_0$$

$$+ \dots$$

$$B = P_0 - P_0 V \frac{Q_0}{a^2} V P_0 + \dots$$

$$A = P_0 V P_0 + P_0 V \frac{Q_0}{a} V P_0 + \dots$$

A segundo orden

Caso $\boxed{E_0^{(0)} = 0}$

$$\Rightarrow \begin{cases} A = \dfrac{S}{2} - \dfrac{1}{2}\left(\dfrac{S}{4}\right)^2 \\ B = 1 - \dfrac{1}{8}\left(\dfrac{S}{4}\right)^2 \end{cases} \qquad A = \frac{S}{2} - \frac{1}{2}\left(\frac{S}{4}\right)^2$$

$$\boxed{P_0 V|\alpha\rangle = E_\alpha|\alpha\rangle_0}$$

$$\boxed{\mathcal{U} P_0 V|\alpha\rangle = \mathcal{U} V|\alpha\rangle}$$

ya que $\mathcal{U} P_0 = \mathcal{U}$, $\mathcal{U}|\alpha\rangle_0 = |\alpha\rangle$

$$\Rightarrow \boxed{\mathcal{U} V|\alpha\rangle = E_\alpha|\alpha\rangle}$$

b) como $(H - E_0)|\alpha\rangle = E_\alpha|\alpha\rangle$ resulta

$$(H - E_0 - \mathcal{U}V)|\alpha\rangle = 0$$

de donde

$$\sum_\alpha (H - E_0 - \mathcal{U}V)|\alpha\rangle_0\langle\alpha| = 0$$

$$\Rightarrow \quad (H - E_0)\mathcal{U} - \mathcal{U}V\mathcal{U} = 0$$

$$\Rightarrow \quad \mathcal{U} = (P_0 + Q_0)\mathcal{U} = P_0\mathcal{U} + Q_0\mathcal{U} = P_0 + Q_0\mathcal{U}$$

$$\underbrace{(H_0 - E_0)P_0}_{=0} + \underbrace{(H_0 - E_0)}_{\substack{\text{Existe inversa} \\ \text{en el espacio} \\ \text{ortogonal a } \mathcal{H}_0}} Q_0\mathcal{U} + V\mathcal{U} - \mathcal{U}V\mathcal{U} = 0$$

$$\Rightarrow \boxed{(E_0 - H_0)\, Q_0\mathcal{U} = V\mathcal{U} - \mathcal{U}V\mathcal{U}}$$

Eligiendo $\dfrac{Q_0}{a}(E_0 - H_0) = 1 \quad \Rightarrow$

$$Q_0\mathcal{U} = \frac{Q_0}{a}(V\mathcal{U} - \mathcal{U}V\mathcal{U})$$

(2)

$$H\psi = \left\{ c_1(t)e^{-iE_1t/\hbar}\tilde{\varepsilon} + c_2(t)e^{-iE_2t/\hbar}\nu \right\}|\mu_1\rangle$$
$$+ \left\{ c_1(t)e^{-iE_1t/\hbar}\nu^* - c_2(t)e^{-iE_2t/\hbar}\tilde{\varepsilon} \right\}|\mu_2\rangle \qquad (1)$$

$$i\hbar\frac{\partial\psi}{\partial t} = i\hbar\left\{ \dot{c}_1 - i\frac{E_1}{\hbar}c_1 \right\}e^{-iE_1t/\hbar}|\mu_1\rangle$$
$$+ i\hbar\left\{ \dot{c}_2 - i\frac{E_2}{\hbar}c_2 \right\}e^{-iE_2t/\hbar}|\mu_2\rangle \qquad (2)$$

$(1)=(2) \Longrightarrow$

$$(i\hbar\dot{c}_1 + E_1c_1) = c_1\tilde{\varepsilon} + c_2e^{-i(E_2-E_1)t/\hbar}\nu$$
$$(i\hbar\dot{c}_2 + E_2c_2) = c_1e^{i(E_2-E_1)t/\hbar}\nu^* - c_2\tilde{\varepsilon}$$

$$\omega_{21} = (E_2-E_1)/\hbar \qquad \Omega = (\omega_{21}+\omega)$$

$$\begin{cases} e^{-i\omega_{21}t}\nu = -\lambda_1e^{-i(\omega_{21}+\omega)t} = -\lambda_1e^{-i\Omega t} \\ e^{i\omega_{21}t}\nu^* = -\lambda_1e^{i(\omega_{21}+\omega)t} = -\lambda_1e^{i\Omega t} \end{cases}$$

$$\boxed{\begin{aligned} i\hbar\dot{c}_1 + E_1c_1 &= c_1\tilde{\varepsilon} - c_2\lambda_1e^{-i\Omega t} \\ i\hbar\dot{c}_2 + E_2c_2 &= -c_2\tilde{\varepsilon} - c_1\lambda_1e^{i\Omega t} \end{aligned}}$$

$$\boxed{\begin{aligned} i\hbar\dot{c}_1 + (E_1-\tilde{\varepsilon})c_1 &= -c_2\lambda_1e^{-i\Omega t} \\ i\hbar\dot{c}_2 + (E_2+\tilde{\varepsilon})c_2 &= -c_1\lambda_1e^{i\Omega t} \end{aligned}}$$

$$\omega_{21} = \frac{(E_2 - E_1)}{\hbar} = -\frac{2\tilde{\varepsilon}}{\hbar}$$

$$\Omega^2 + 4\lambda_1^2/\hbar^2 = \Omega^2 + \gamma^2\hbar^2 B_1^2 = ($$

$$\Omega = \omega_{21} + \omega = -\frac{2\tilde{\varepsilon}}{\hbar} + \omega = \frac{1}{\hbar}(\hbar\omega - 2\tilde{\varepsilon})$$

$$\boxed{\Omega = \frac{1}{\hbar}(\hbar\omega - 2\tilde{\varepsilon})}$$

$$\boxed{c_1(t) = a\, e^{-i\eta_+ t} + b\, e^{-i\eta_- t}} \qquad c_1(0) = 0 \rightarrow a = -b$$

$$c_1(t) = a\left(e^{-i\eta_+ t} - e^{-i\eta_- t}\right) \qquad \Delta = \sqrt{\Omega^2 + 4\lambda_1^2/\hbar^2}$$

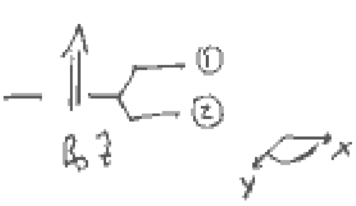
$$\eta_+ = \frac{\Omega}{2} + \frac{\Delta}{2}$$

$$\eta_- = \frac{\Omega}{2} - \frac{\Delta}{2}$$

$$c_1(t) = a\, e^{-i\Omega t/2}\left[e^{-i\Delta t/2} - e^{+i\Delta t/2}\right]$$

$$c_1(t) = a\, e^{-i\Omega t/2}\left[(-2i)\sin\frac{\Delta t}{2}\right]$$

$$\boxed{c_1(t) = -2ia\, e^{-i\Omega t/2}\left[\sin\frac{\Delta t}{2}\right]}$$



$$i\hbar\, \dot{c}_2(t) = -\lambda_1 e^{i\Omega t} a\left(e^{-i\eta_+ t} - e^{-i\eta_- t}\right)$$

$$= -\lambda_1 a\left[e^{i(\Omega-\eta_+)t} - e^{+i(\Omega-\eta_-)t}\right]$$

$$c_2 = \frac{1}{i\hbar}(-\lambda_1 a)\left\{\frac{e^{i(\Omega-\eta_+)t}}{i(\Omega-\eta_+)} - \frac{e^{i(\Omega-\eta_-)t}}{i(\Omega-\eta_-)}\right\}$$

$$c_2(t) = \left(\frac{\cancel{i}\lambda_1 a}{\hbar}\right)\left\{\frac{e^{i(\Omega-\eta_+)t}}{(\Omega-\eta_+)} - \frac{e^{i(\Omega-\eta_-)t}}{(\Omega-\eta_-)}\right\}$$

$$c_2(t=0) = \lambda_1\frac{a}{\hbar}\left\{\frac{\Omega-\eta_- - \Omega + \eta_+}{(\Omega-\eta_+)(\Omega-\eta_-)}\right\}$$

(J)

$\{\ \}$

$$= \frac{\Omega_+ - \Omega_-}{(\Omega-\Omega_+)(\Omega-\Omega_-)} = \frac{\Delta}{\Omega_- \Omega_+}$$

$$= \left[\frac{4\Delta}{(\Omega^2-\Delta^2)}\right]$$

$$= \frac{4\Delta}{-4\lambda_1^2/\hbar^2} = -\left(\frac{\Delta\hbar^2}{\lambda_1^2}\right)$$

$$c_2(t=0) = 1 \Rightarrow \frac{\cancel{\lambda_1} a}{\cancel{\hbar}}(-)\frac{\Delta\hbar^{\cancel{2}}}{\lambda_1^{\cancel{2}}}$$

$$= -\frac{a\Delta\hbar}{\lambda_1} = 1$$

$$a = \left(-\frac{\lambda_1}{\Delta\hbar}\right)$$

$$c_1(t) = (-2i)\left(-\frac{\lambda_1}{\Delta\hbar}\right)e^{-i\Omega t/2}\sin\left(\frac{\Delta t}{2}\right)$$

$$\boxed{c_1(t) = \left(\frac{2\lambda_1 i}{\Delta\hbar}\right)e^{-i\Omega t/2}\sin\left(\frac{\Delta t}{2}\right)}$$

$$c_2(t) = -\frac{\lambda_1^2}{\Delta\hbar^2}\left[\frac{e^{i\Omega_- t}}{\Omega_-} - \frac{e^{i\Omega_+ t}}{\Omega_+}\right]$$

$$= -\frac{\lambda_1^2}{\Delta\hbar^2}e^{i\Omega t/2}\left\{\frac{e^{-i\Delta t/2}}{\Omega_-} - \frac{e^{i\Delta t/2}}{\Omega_+}\right\}$$

$$= (-)\frac{\lambda_1^2}{\Delta\hbar^2}e^{i\Omega t/2}\left\{\frac{\Omega}{2}\left(e^{-i\Delta t/2} - e^{i\Delta t/2}\right) + \frac{\Delta}{2}\left(e^{-i\Delta t/2} + e^{i\Delta t/2}\right)\right\}\cdot\frac{1}{\Omega\Omega_+}$$

de desde como $\rho_0 + \rho_S = 1$

$$\Rightarrow (1 - \rho_0)\mathcal{U} = \mathcal{U} - \rho_0\mathcal{U} = \mathcal{U} - \rho_0$$

$$\Rightarrow \mathcal{U} = \rho_0 + \frac{\Phi_0}{a}(\nabla\mathcal{U} - \mathcal{U}\nabla\mathcal{U})$$

Desarrollo

$$\mathcal{U} = \mathcal{U}^{(0)} + \mathcal{U}^{(1)} + \cdots$$

$$\mathcal{U}^{(0)} = \rho_0$$

$$\cancel{\rho_0} + \mathcal{U}^{(1)} = \cancel{\rho_0} + \frac{\Phi_0}{a}(\nabla\rho_0 + \rho_0\nabla\rho_0)$$

$$\mathcal{U}^{(1)} = \frac{\Phi_0}{a}\nabla\rho_0 - \underbrace{\left(\frac{\Phi_0}{a}\rho_0\right)}_{\approx 0}\nabla\rho_0$$

$$\mathcal{U}^{(1)} = \frac{\Phi_0}{a}\nabla\rho_0$$

etc

$$\{\ \} = \frac{\Omega}{2}(-2i)\sin\frac{\Delta t}{2} + \frac{\Delta}{2}\, 2\cos\frac{\Delta t}{2}$$

$$= \Delta\cos\left(\frac{\Delta t}{2}\right) - i\Omega\sin\left(\frac{\Delta t}{2}\right)$$

$$c_2(t) = -\frac{\lambda_1^2}{\hbar^2\Delta} e^{i\Omega t/2}\left(\Delta\cos\frac{\Delta t}{2} - i\Omega\sin\frac{\Delta t}{2}\right)\frac{1}{\frac{(-4\lambda_1^2/\hbar^2)}{4}}$$

$$= \frac{e^{i\Omega t/2}}{\Delta}\left(\Delta\cos\frac{\Delta t}{2} - i\Omega\sin\frac{\Delta t}{2}\right)$$

$$\boxed{c_2(t) = e^{\frac{i\Omega t}{2}}\left\{\cos\frac{\Delta t}{2} - i\frac{\Omega}{\Delta}\sin\frac{\Delta t}{2}\right\}}$$

$$c_2(t=0) = 1$$
$$c_1(t=0) = 0$$

$$\left[\frac{\lambda_1^2}{\hbar^2}t^2 \times \left(\frac{\sin^2\alpha}{\alpha^2}\right)\right]$$

$$|c_1|^2_t = \frac{4\lambda_1^2}{\Delta^2\hbar^2}\cdot\sin^2\left(\frac{\Delta t}{2}\right) = \frac{\lambda_1^2}{\hbar^2}t^2\left[\frac{\sin^2(\Delta t/2)}{(\Delta t/2)^2}\right]$$

$$|c_2|^2_t = \cos^2\left(\frac{\Delta t}{2}\right) + \frac{\Omega^2}{\Delta^2}\sin^2\left(\frac{\Delta t}{2}\right)$$

$$|c_1|^2_t + |c_2|^2_t = \cos^2\left(\frac{\Delta t}{2}\right) + \sin^2\left(\frac{\Delta t}{2}\right)\left\{\frac{\Omega^2}{\Delta^2} + \frac{4\lambda_1^2}{\hbar^2\Delta^2}\right\}$$

$$\frac{1}{\Delta^2}\left(\Omega^2 + \frac{4\lambda_1^2}{\hbar^2}\right) = \frac{\Delta^2}{\Delta^2} = 1$$

$$= 1 \quad !$$

$$\boxed{|c_1|^2_t = \frac{\lambda_1^2}{\hbar^2}t^2\left[f(\Delta t/2)\right]} \qquad \boxed{f(\Delta t/2) = \frac{\sin^2(\Delta t/2)}{(\Delta t/2)^2}}$$

$$\boxed{i\hbar \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \end{bmatrix} = \begin{bmatrix} (\tilde{\varepsilon} - E_1) & -\lambda_1 e^{-i\Omega t} \\ -\lambda_1 e^{i\Omega t} & -(\tilde{\varepsilon} + E_2) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}$$ ③

②

$$X = M \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad M = M(t) \quad \exists\, M^{-1}$$

$$M M^{-1} = \mathbb{1}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = (M^{-1} X)$$

$$i\hbar \frac{d}{dt} \left[M^{-1} X \right] = \left(U(t)\, M^{-1}(t) \right) X$$

$$i\hbar\, M \frac{d}{dt} \left(M^{-1} X \right) = \left(M U M^{-1} \right) X$$

$$\boxed{M U M^{-1} = \text{diagonal}} = A(t)$$

$$\frac{d}{dt} \left[M M^{-1} X \right] = \dot{M} \left(M^{-1} X \right) + \boxed{M \frac{d}{dt} \left(M^{-1} X \right)}$$

$$i\hbar \left\{ \frac{d}{dt} [X] - \dot{M} \left(M^{-1} X \right) \right\} = A(t) X$$

$$\boxed{i\hbar \frac{d}{dt} [X] - i\hbar \left(\dot{M} M^{-1} \right) [X] = A(t) [X]}$$

$$S_x |1/2, m_s\rangle = \hbar \left[\frac{3}{4} - m_s(m_s+1)\right]^{1/2} |1/2, m_s+1\rangle$$

$$|\alpha\rangle = |1/2\ 1/2\rangle \qquad \beta = |1/2\ -1/2\rangle$$

$$\langle\alpha|\alpha\rangle = \langle\beta|\beta\rangle = 1$$

$$\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle = 0$$

$$[S_\alpha, S_\beta] = i\hbar S_\gamma \left\{ sg\ P\begin{pmatrix} x & y & z \\ \alpha & \beta & \gamma \end{pmatrix} \right\}$$

$$S_x|\alpha\rangle = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\hbar}{2}|\beta\rangle$$

$$S_x|\beta\rangle = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}|\alpha\rangle$$

$$S_y|\alpha\rangle = \frac{i\hbar}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{i\hbar}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = i\frac{\hbar}{2}|\beta\rangle$$

$$S_y|\beta\rangle = \frac{i\hbar}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{i\hbar}{2}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = -i\frac{\hbar}{2}|\alpha\rangle$$

$$S_z|\alpha\rangle = \frac{\hbar}{2}|\alpha\rangle$$

$$S_z|\beta\rangle = -\frac{\hbar}{2}|\beta\rangle$$

Normalización

$$|\chi\rangle = a_1|\alpha\rangle + a_2|\beta\rangle$$

$$a_1 = \langle\alpha|\chi\rangle$$

$$a_2 = \langle\beta|\chi\rangle$$

$$|\chi\rangle = \langle\alpha|\chi\rangle|\alpha\rangle + \langle\beta|\chi\rangle|\beta\rangle$$

$$\langle\chi|\chi\rangle = \underbrace{|\langle\alpha|\chi\rangle|^2}_{\text{partícula en eje ↑ "up"}} + |\underbrace{\langle\beta|\chi\rangle}_{\text{partícula en eje ↓ "down"}}|^2 = |a_1|^2 + |a_2|^2$$

"Medidas del spin en direc. pre-determinada"

Dado

$|\chi\rangle = a_1|\alpha\rangle + a_2|\beta\rangle$

Consideremos un vector $\hat{n}$ (unitario), tal que

$$\boxed{\hat{n}\cdot\hat{S}\,|\chi\rangle = \frac{\hbar}{2}\lambda\,|\chi\rangle}$$

Supongamos que $\hat{n}$ está en el plano (x, z)

$\hat{n} \equiv (\sin\theta, 0, \cos\theta)$

$$\Rightarrow \boxed{\hat{n}\cdot\bar{S} = S_x \sin\theta + S_z\cos\theta}$$

$$\Rightarrow \hat{n}\cdot\bar{S}\,|\chi\rangle = \sin\theta\, a_1\, S_x|\alpha\rangle + \cos\theta\, a_1\, S_z|\alpha\rangle + \sin\theta\, a_2\, S_x|\beta\rangle + \cos\theta\, a_2\, S_z|\beta\rangle$$

$$= \sin\theta\, a_1 \frac{\hbar}{2}|\beta\rangle + \cos\theta\, a_1\frac{\hbar}{2}|\alpha\rangle + \sin\theta\, a_2\frac{\hbar}{2}|\alpha\rangle + \cos\theta\, a_2\left(-\frac{\hbar}{2}\right)|\beta\rangle$$

$$= \frac{\hbar\lambda}{2}\left(a_1|\alpha\rangle + a_2|\beta\rangle\right)$$

$$\Rightarrow |\beta\rangle\left(a_1\frac{\hbar}{2}\sin\theta - a_2\frac{\hbar}{2}\cos\theta\right) + |\alpha\rangle\left(a_1\frac{\hbar}{2}\cos\theta + a_2\frac{\hbar}{2}\sin\theta\right)$$

$$= \left(\frac{\hbar\lambda}{2}a_1\right)|\alpha\rangle + \left(\frac{\hbar\lambda}{2}a_2\right)|\beta\rangle$$

per lo tanto $\rightarrow$

$$\boxed{\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \lambda \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}}$$

Soluzione

$$\mathrm{Det}\left[\begin{pmatrix} \cos\theta - \lambda & \sin\theta \\ \sin\theta & -\cos\theta - \lambda \end{pmatrix}\right] = 0$$

$$\Rightarrow -(\cos\theta - \lambda)(\cos\theta + \lambda) - \sin^2\theta = 0$$

$$\cos^2\theta - \lambda^2 + \sin^2\theta = 0$$

$$\cos^2\theta + \sin^2\theta = \lambda^2$$

$$\boxed{\lambda = \pm 1}$$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

Solo $\longrightarrow$ $\boxed{\lambda = 1}$

$$\cos\theta\, a_1 + \sin\theta\, a_2 = a_1$$

$$a_1(1 - \cos\theta) = + a_2 \sin\theta$$

$$2 a_1 \sin^2(\theta/2) = + a_2 \sin(\theta/2 + \theta/2)$$

$$\cancel{2} a_1 \sin^2\theta/2 = + \cancel{2} a_2 \sin\theta/2 \cos\theta/2$$

$$\boxed{a_1 \sin\theta/2 = a_2 \cos\theta/2} \quad (\lambda = 1)$$

$\lambda = -1$

analogamente $\cos\theta\, a_1 + \sin\theta\, a_2 = -a_1$

$$\sin\theta\, a_2 = -(1 + \cos\theta)\, a_1$$

$$\cancel{2} \sin\theta/2 \cancel{\cos\theta/2}\, a_2 = -a_1 \cancel{2} \cos^2\theta/2$$

$$\Rightarrow \boxed{(\sin\theta/2)\, a_2 = -(\cos\theta/2)\, a_1 \quad (\lambda = -1)}$$

$$i\hbar \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\lambda_1 e^{-i\Omega t} \\ -\lambda_1 e^{i\Omega t} & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\boxed{\tilde{\varepsilon} = E_1 = -E_2}$$

$$i\hbar \dot{c}_1 = -\lambda_1 e^{-i\Omega t} c_2$$

$$i\hbar \dot{c}_2 = -\lambda_1 e^{i\Omega t} c_1$$

$$i\hbar \ddot{c}_1 = i\lambda_1 \Omega e^{-i\Omega t} c_2 - \lambda_1 e^{-i\Omega t} \dot{c}_2$$

$$i\hbar \ddot{c}_1 = -i\Omega (i\hbar \dot{c}_1) - \lambda_1 e^{-i\Omega t} \left(\frac{i}{\hbar} \lambda_1 e^{i\Omega t}\right) c_1$$

$$\boxed{i\hbar \ddot{c}_1 - \Omega \hbar \dot{c}_1 + \frac{\lambda_1^2}{\hbar} i c_1 = 0}$$

$$c_1 \simeq e^{-i\eta t}$$

$$\dot{c}_1 \simeq -i\eta c_1$$

$$\ddot{c}_1 = -\eta^2 c_1$$

$$\left[i\hbar(-\eta^2) - \Omega\hbar(-i\eta) + \frac{\lambda_1^2 i}{\hbar}\right] c_1 = 0$$

$$\Rightarrow \left(-\eta^2 + \Omega\eta + \frac{\lambda_1^2}{\hbar^2}\right) = 0$$

$$\eta^2 - \Omega\eta - \frac{\lambda_1^2}{\hbar^2} = 0$$

$$\boxed{\eta = \frac{1}{2}\left[\Omega \pm \sqrt{\Omega^2 + 4\lambda_1^2/\hbar^2}\right]}$$

$$\dot{M} M^{-1} = B(t)$$

$$\frac{d}{dt}\left[X\right] = \left\{\left(-\frac{i}{\hbar}\right) A(t) + B(t)\right\} X$$

$$\boxed{F(t) = -\frac{i}{\hbar} A(t) + B(t)}$$

$$\frac{d}{dt}\left[X\right] = F(t) X$$

$$\frac{d}{dt}\left(\lg X\right) = F(t)$$

$$X(t) = X(0)\, e^{\int^{t} F(t)\, dt'}$$

$$\boxed{X(t) = X(0) \exp\left[\int^{t} F(z)\, dz\right]}$$